

MATHEMATICS

A Textbook for Class VIII

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$$\frac{12 \times 2}{6 \times 2} = \frac{24}{12}$$

$$\frac{15 \times 2}{26} = \frac{30}{26}$$

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G. HARISH





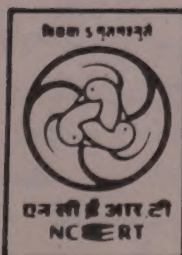
MATHEMATICS

A Textbook for Class VIII

Part I

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राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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Foreword

The National Policy on Education (NPE) 1986 has emphasised the need for qualitative improvement of school education, particularly in the area of science and mathematics education. The Government of India has already initiated a number of steps in this direction: The National Council of Educational Research and Training (NCERT) has been assigned the responsibility of developing a new curriculum and related curricular materials in line with the new education policy to serve as models for the States and the Union Territories to adopt/adapt.

The NCERT has been working in the area of curriculum development/renewal for many years. Yet, developing a curriculum in tune with the intentions and aspirations of the NPE poses a real challenge. The various curricular issues arising out of the NPE were discussed in a number of seminars/workshops resulting in the Council's document *National Curriculum for Elementary and Secondary Education — A Framework* (Revised version 1987) and several other documents.

Science and mathematics are the vital areas of school curriculum. So, the NCERT thought it appropriate to make the best use of the available expertise in the country in science and mathematics education in the process of curriculum renewal and preparation of new generation of instructional materials. Accordingly, a General Advisory Board for Science and Mathematics was constituted under the chairmanship of Professor C.N.R. Rao, an eminent scientist and Chairman of the Prime Minister's Science Advisory Council. On the advice of this General Advisory Board, six writing teams were set up for developing new instructional packages in science and mathematics in line with the NPE. The writing team for mathematics was constituted under the chairmanship of Prof. U.N. Singh. The team consists of distinguished mathematicians from various universities, besides NCERT experts. The writing team met several times and after much deliberations evolved a new mathematics curriculum which, I am sure reflects the intentions and aspirations of the NPE. In the previous year, the writing team developed the textbook for Class VII based on the new syllabus. The present textbook for Class VIII is in continuation of the textbook for Class VII.

The authors spared no efforts to produce materials of high quality. First, the draft materials prepared by different authors were continuously refined through mutual discussions within the group. Then the materials were exposed to a group of classroom teachers drawn from all over the country in a review workshop. The suggestions and comments made in the review workshop were incorporated by the authors as far as possible and finally, the whole manuscript was edited by Prof. J.N. Kapur.

I am indeed very thankful to Prof. J.N. Kapur, Prof. D.K. Sinha, Prof. M.K. Singal, Prof. P.D. Gupta, Prof. Asha Rani Singal, Prof. J.D. Gupta, Dr S.P. Arya, Dr Sunder Lal, Dr Surja Kumari and Shri Mahendra Shanker who authored different parts of the book. I am particularly grateful to Prof. J.N. Kapur who provided valuable guidance to his team of authors and finally edited the whole manuscript. I thank Prof. U.N. Singh, Chairman of the writing team for his guidance in the development of this book. I express my deep appreciation to my colleagues in the Department of Education in Science and Mathematics, Dr Surja Kumari, Shri Mahendra Shanker and Dr (Ms) Mukti Acharya who took a lot of pains in shaping the manuscript in the pressworthy form and seeing it through the press. I am very much indebted to the teachers who participated in the review workshop and provided valuable suggestions and comments for the improvement of the draft manuscript. I thank Prof. B. Ganguly, Head, Department of Education in Science and Mathematics who took a lot of interest in this project and greatly helped in bringing out this book. I also express my thanks to Shri C.N. Rao, Head, Publication Department and his Publication Team for taking all pains in bringing out this book expeditiously in an excellent form.

Curriculum development is a dynamic and continuous process. No one can claim to have developed a perfect curriculum or perfect curricular materials. Though Prof. U.N. Singh and his able team of mathematicians did a very good job of evolving a new mathematics curriculum and new curricular materials in line with the NPE, there will always be some scope for further improvement. So, I request all those who will be using this book to offer their valuable suggestions for further improvement.

P.L.MALHOTRA

Director

National Council of Educational
Research and Training

Preface

As I had said in the Preface to the textbook for Class VII, the new series of textbooks in mathematics being prepared by the NCERT has been written in the light of paragraphs 8.16 and 8.17 of the National Policy on Education 1986. These paragraphs are quoted in full in the Preface to the textbook for Class VII. The textbook for Class VIII has been written in continuation of the textbook for Class VII. Most of the topics introduced in the book for Class VII have been further studied in the present textbook as a logical development of the subject matter studied in Class VII. I reproduce below a part of my Preface to the textbook for Class VII with slight modifications, because the points emphasized there are equally applicable to the present textbook.

The implementation of the NPE made it necessary to review the courses of studies in science and mathematics and to bring out new textbooks. There is another strong reason for reviewing the course contents in science and mathematics and for rewriting textbooks in these disciplines. Different branches of science, pure and applied, including mathematics, are developing with astonishing rapidity. Exciting discoveries of far-reaching importance are being made in quick succession. Deep and new ideas of a rapidly growing science very often shed new and penetrating light even on the most elementary topics. It is, therefore, highly desirable that courses of studies of school education are reviewed periodically and new textbooks are written.

The National Council of Educational Research and Training initiated prompt action in respect of redesigning the curricula in science subjects and mathematics relating to school education. It appointed a General Advisory Board for Science and Mathematics under the Chairmanship of Professor C.N.R. Rao, who is also the Chairman of the Science Advisory Council of the Prime Minister. On the advice of General Advisory Board, the NCERT constituted six writing teams for developing instructional packages in science subjects and mathematics from upper primary to senior secondary level. Besides the experts from the NCERT, distinguished mathematicians from different parts of the country are members of the writing team in mathematics.

The NCERT had done a good deal of preparatory work in connection with curriculum renewal before the appointment of the writing team. We have been greatly benefited in our work by the NCERT's documents:

- National Curriculum for Elementary and Secondary Education – A Framework
- Mathematics education for the first 10 years of schooling – Guidelines for developing curriculum for upper primary and high school stages
- Draft syllabi in mathematics for upper primary, secondary and senior secondary levels

This book contains more detailed explanation of concepts, more number of illustrations, solved examples and exercises. This is done purposely so that pupils can acquire better understanding of concepts, and better problem-solving skills. A number of learning activities have been suggested for (re)discovering many mathematical facts and for developing drawing and measuring skills. These and more such learning activities should be carried out in the classroom/mathematics workshop/open field. Some of the word problems are structured around themes such as the socio-economic development in the country, need for protection of environment, need for removal of social barriers, need for observance of small family norms, elimination of sex bias, etc. The silent messages of these word problems should get into the mind of the pupils, and the teachers must be conscious of this while teaching.

To achieve the objectives said in the NPE a lot of work has to be done, particularly in the direction of ensuring minimum levels of learning for all, as well as nurturing the high achievers/talented. The group is working on the development of additional instructional material to supplement this textbook. The additional materials are Supplementary Problem Book, Enrichment Mathematics, Teachers' Guide, and so on. I hope these additional materials will soon be made available to the students and teachers.

This book is developed by the team consisting of Prof. J.N. Kapur, Prof. M.K. Singal, Prof. Asha Rani Singal, Prof. P.D. Gupta, Prof. J.D. Gupta, Prof. D.K. Sinha, Dr S.P. Arya, Dr Sunder Lal, Dr Surja Kumari and Shri Mahendra Shanker. The draft manuscript was first exposed to a group of classroom teachers from all over the country in a Review Workshop organised by the NCERT at New Delhi and in the light of the various suggestions/comments made by this group of teachers, the authors revised their draft materials. Finally, the manuscript was edited by Prof. J.N. Kapur who is the Convenor of this group. I am thankful to Prof. J.N. Kapur and members of his team for the excellent work done by them.

I am thankful to Dr P.L. Malhotra, Director, NCERT who initiated this project and gave us an opportunity to join this national endeavour for the improvement of mathematics education.

I am grateful to Prof. A.K. Jalaluddin, Joint Director, NCERT and to Prof. B. Ganguly, Head, Department of Education in Science and Mathematics for the cooperation extended by them in this work.

My sincere thanks are also due to all the teachers who participated in the Review Workshop and provided us with their valuable suggestions and comments. If the present series of the textbooks contributes to the improvement of the quality of education in mathematics in schools, our efforts will be amply remunerated.

Dr Surja Kumari (Coordinator), Shri Mahendra Shanker, Dr Mukti Acharya of NCERT had to put in a lot of hard work in organising several workshops, getting the manuscript in pressworthy form and finally seeing it through the press. I am indeed very thankful to them.

In spite of the great care taken by the authors and the NCERT team, some errors may have escaped our notice. Suggestions for improving the quality of the book will be gratefully received.

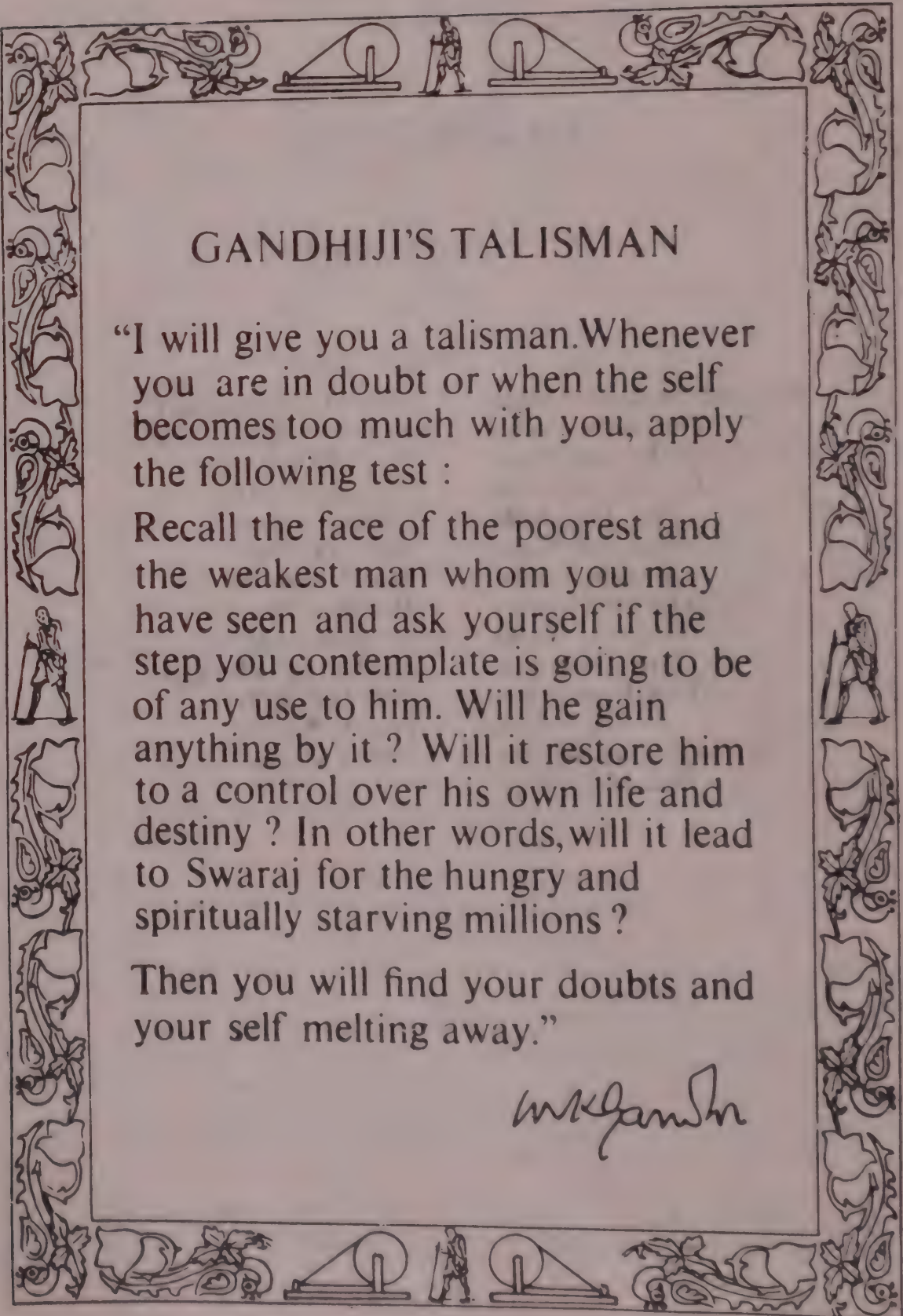
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GANDHIJI'S TALISMAN

“I will give you a talisman. Whenever you are in doubt or when the self becomes too much with you, apply the following test :

Recall the face of the poorest and the weakest man whom you may have seen and ask yourself if the step you contemplate is going to be of any use to him. Will he gain anything by it ? Will it restore him to a control over his own life and destiny ? In other words, will it lead to Swaraj for the hungry and spiritually starving millions ?

Then you will find your doubts and your self melting away.”

M.K. Gandhi

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SCIENCE RELATED VALUES

Curiosity, quest for knowledge, objectivity, honesty and truthfulness, courage to question, systematic reasoning, acceptance after proof/verification, open-mindedness, search for perfection and team spirit are some of the basic values related to science. The processes of science, which help in searching the truth about nature and its phenomena are characterised by these values. Science aims at explaining things and events. Therefore to learn and practise science :

- * Be inquisitive about things and events around you.
- * Have the courage to question beliefs and practices.
- * Ask 'what', 'how' and 'why' and find your answers by critically observing, experimenting, consulting, discussing and reasoning.
- * Record honestly your observations and experimental results in your laboratory or outside it.
- * Repeat experiments carefully and systematically if required, but do not manipulate your results under any circumstance.
- * Be guided by facts, reasons and logic. Do not be biased in one way or the other.
- * Aspire to make new discoveries and inventions by sustained and dedicated work.

Review Exercises on Work Done in Class VII

[UNLESS OTHERWISE STATED, a rational number means a number of the form $\frac{p}{q}$ where p, q are integers, and $q \neq 0$.]

1. Which one of the following is not an integer?

(a) $-16 + 0$

(b) $9 - 11$

(c) -5×9

(d) $37 \div 9$

2. Which one of the following is not meaningful? Which ones of the rest would produce a non-integer rational number?

(a) $8 \div 2$

(b) $8 \div 3$

(c) $8 \div 16$

(d) $8 \div 0$

(e) $0 \div 8$

(f) $8 \div 8$

3. What are the numerators of the following rational numbers?

$$\frac{3}{7}, \frac{-5}{9}, \frac{7}{-11}, \frac{6}{1}, \frac{0}{1}, \frac{9}{3}$$

Also, make a list of the denominators.

4. Pick out the positive rational numbers from amongst the following :

$$\frac{2}{3}, \frac{-2}{3}, \frac{2}{-3}, \frac{-2}{-3}$$

5. Give five examples of non-integer rational numbers.

6. Write five rational numbers which happen to be integers.

7. When is a rational number said to be in the standard form? Write the following rational numbers in their standard form:

$$\frac{6}{14}, \frac{10}{140}, \frac{-9}{3}, \frac{3}{-7}, \frac{-5}{-9}$$

8. Pick out that rational number from the following list which is not equal to $\frac{3}{5}$:

$$\frac{-3}{5}, \frac{-3}{-5}, \frac{3}{5}, \frac{6}{10}, \frac{-12}{-20}, \frac{30}{50}$$

9. Write each of the following numbers as a mixed fraction :

$$\frac{31}{4}, \frac{27}{6}, \frac{43}{5}, \frac{121}{8}, \frac{-88}{13}, \frac{-43}{5}$$

[Hint : Simplify $\frac{27}{6}$ before writing it as a mixed fraction.]

10. Write each of the following rational numbers in the standard form :

$$3\frac{4}{5}, 6\frac{2}{3}, -5\frac{1}{4}, -7\frac{2}{3}$$

11. Represent the rational numbers $\frac{1}{2}$, $\frac{-3}{4}$, $\frac{3}{2}$ and $\frac{5}{8}$ on the number line.

12. For each of the rational numbers given in problems 9 and 10, find the two consecutive integers between which it lies.

13. Find, in each of the following, whether the given numbers are equal :

(a) $\frac{3}{7}, \frac{-3}{-7}$

(b) $\frac{5}{9}, \frac{100}{180}$

(c) $\frac{21}{57}, \frac{42}{114}$

(d) $\frac{6}{7}, \frac{36}{37}$

(e) $\frac{11}{25}, \frac{110}{250}$

(f) $\frac{3679}{4182}, \frac{-25437}{33646}$

14. Write $\frac{2}{9}$ in five different forms. For example, $\frac{-4}{-18}$.

15. Write $\frac{1}{2}$ with denominator as

$$10, 16, 26, 102, 5000.$$

Can you write $\frac{1}{2}$ with denominator equal to 5 ?

16. Write $\frac{5}{8}$ with numerator as

$$10, 35, 80, 1200, 5000.$$

Can you make the numerator equal to 13 ?

17. Which of the following are not in their lowest terms ?

$$\frac{11}{15}, \frac{17}{119}, \frac{19}{95}, \frac{21}{85}, \frac{23}{105}$$

18. Express each of the following in the lowest terms :

$$\frac{4}{8}, \frac{10}{30}, \frac{11}{55}, \frac{13}{65}, \frac{24}{96}, \frac{15}{10}, \frac{32}{6}$$

19. Arrange the rational numbers $\frac{3}{7}, \frac{-5}{9}, \frac{-7}{-11}, \frac{6}{1}, \frac{0}{1}$

and $\frac{9}{3}$ in the ascending order. Also, arrange $3\frac{4}{5}, 6\frac{2}{3}, -7\frac{2}{3}$ and $-5\frac{1}{4}$ in the descending order.

20. Add and write the sum in lowest terms :

(a) $\frac{2}{3} + \frac{7}{6} + \frac{1}{12}$

(b) $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$

(c) $\frac{7}{5} + \frac{11}{15} + \frac{13}{20}$

(d) $\frac{4}{7} + \frac{21}{70} + \frac{3}{35}$

21. Rearrange suitably and find the sum :

(a) $\frac{2}{5} + \frac{5}{6} + \frac{8}{5} + \frac{1}{6}$

(b) $\frac{4}{3} + \frac{1}{4} + \frac{16}{9} + \frac{3}{16}$

(c) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{9}$

(d) $\frac{2}{7} + \frac{3}{8} + \frac{4}{21} + \frac{9}{16}$

22. Write the negative of each of the following rational numbers :

$$\frac{2}{3}, \frac{-4}{-5}, \frac{-3}{5}, \frac{-7}{-8}, -\frac{6}{7}, 0$$

23. Subtract :

(a) $\frac{2}{5}$ from $\frac{4}{5}$

(b) $\frac{2}{3}$ from $\frac{4}{5}$

(c) $\frac{4}{5}$ from $\frac{1}{6}$

(d) $\frac{1}{5}$ from 2

(e) $\frac{3}{7}$ from -1

(f) $\frac{7}{9}$ from 0

24. Simplify :

(a) $\frac{2}{3} + \frac{4}{5} - \frac{1}{6}$

(b) $\frac{1}{7} - \frac{3}{14} + \frac{5}{7}$

(c) $\frac{1}{4} - \frac{3}{4} - \frac{7}{8}$

(d) $3 + \frac{1}{2} - 6\frac{1}{4}$

25. Evaluate :

(a) $\frac{3}{4} + \frac{3}{4} + \dots$ ten times.

(b) $\frac{-5}{6} + \frac{-5}{6} + \dots$ twenty times

(c) $2\frac{3}{4} + 2\frac{3}{4} + \dots$ thirty times

(d) $\left(-3\frac{4}{5}\right) + \left(-3\frac{4}{5}\right) + \dots$ forty times

26. Express the product in lowest terms :

(a) $\frac{4}{5} \times \frac{7}{8}$

(b) $\frac{2}{3} \times \frac{6}{11}$

(c) $\frac{-3}{5} \times \frac{35}{27}$

(d) $\frac{4}{11} \times \left(\frac{-1}{6}\right)$

27. Multiply :

(a) $1 \times \frac{7}{8}$

(b) $1 \times \left(-\frac{6}{7}\right)$

(c) $-1 \times \frac{4}{7}$

(d) $-1 \times \left(-\frac{8}{7}\right)$

(e) $0 \times \frac{2}{5}$

(f) $\left(-\frac{7}{5}\right) \times 0$

28. Use the distributivity of multiplication over addition and simplify :

(a) $\frac{5}{3} \left(\frac{24}{5} + \frac{9}{10}\right)$

(b) $\frac{4}{5} \left(\frac{5}{8} + \frac{15}{16}\right)$

(c) $\frac{5}{2} \left(\frac{16}{7} - \frac{4}{21}\right)$

(d) $\frac{4}{3} \left(\frac{9}{8} - \frac{15}{4}\right)$

29. Find the reciprocals of the following rational numbers :

$7, \frac{7}{13}, \frac{31}{9}, 6\frac{2}{5}$

30. Divide:

(a) 1 by $\frac{7}{8}$

(b) 2 by $\left(-\frac{6}{7}\right)$

(c) $\frac{4}{3}$ by $\frac{5}{3}$

(d) $\frac{8}{7}$ by $\left(-\frac{5}{7}\right)$

(e) $\frac{6}{5}$ by 1

(f) 0 by $\frac{6}{7}$

31. Simplify :

$$(a) \frac{4}{11} + \frac{22}{5} \times \frac{5}{4}$$

$$(b) \frac{2}{7} \times \frac{14}{6} + \frac{4}{3}$$

$$(c) \frac{3}{7} + \frac{4}{5} \div \frac{7}{10}$$

$$(d) \frac{3}{4} - \frac{2}{3} + \frac{8}{9}$$

[Use the same rules as for simplifying expressions involving integers alone.]

32. Find three rational numbers between :

$$(a) \frac{1}{2} \text{ and } \frac{1}{4}$$

$$(b) \frac{2}{3} \text{ and } \frac{5}{6}$$

33. Write the absolute value of :

$$4, -23, \frac{47}{13}, \frac{-3}{10}, \frac{-21}{43}$$

34. Write as a decimal :

$$\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{3}{2}, \frac{5}{4}, \frac{7}{8}, \frac{3}{5}, \frac{1}{3}, \frac{4}{9}, \frac{19}{90}$$

35. Express in the form $\frac{p}{q}$:

$$2.\bar{1}, 3.\bar{2}, 7.\bar{23}, 7.2\bar{3}, 8.\bar{15}$$

36. Is $2.3 = 2.30$? What about $2.\bar{3}$ and $2.\bar{30}$?

37. Express in the exponential form :

$$(a) \frac{2}{3} \times \frac{2}{3}$$

$$(b) \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$$

$$(c) \frac{-5}{3} \times \frac{-5}{3} \times \frac{-5}{3}$$

$$(d) \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)$$

$$(e) \frac{16}{81}$$

38. Express in the form $\frac{p}{q}$:

$$\left(\frac{2}{7}\right)^2, \left(\frac{3}{4}\right)^3, \left(-\frac{7}{6}\right)^2, \left(-\frac{3}{5}\right)^3$$

39. Simplify :

$$(a) \left(\frac{2}{5}\right)^2 \times \left(\frac{2}{5}\right)^3$$

$$(b) \left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^2$$

$$(c) \left(\frac{3}{5}\right)^{22} + \left(\frac{3}{5}\right)^{33}$$

$$(d) \left(\frac{7}{9}\right)^3 + \left(\frac{7}{9}\right)^3$$

(e) $\left[\left(\frac{4}{7}\right)^{-3}\right]^0$ (f) $\left[\left(\frac{4}{3}\right)^2\right]^3$

40. Express in the scientific notation :

123.456 , 2345.678 , 0.45678 , 0.0076543

41. Express each of the following as a decimal :

1.311×10^2 , 6.51205×10^4 , 6.123×10^{-3} , 7.4612×10^{-4}

42. Find whether the variation amongst x and y is direct, inverse or neither :

(a)

x	2	4	8	16	32
y	32	16	8	4	2

(b)

x	3	5	7	9	11
y	9	15	21	27	33

(c)

x	1	3	5	7	9
y	1	3	5	7	9

(d)

x	2	3	4	5	6
y	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$

(e)

x	2	3	4	5	6
y	3	4	5	6	7

43. Ramu can reap a field in 20 days. What part of the field would he have reaped in 15 days ?

44. Nandu can polish a table in 6 hours. Jalaluddin can polish this table in 4 hours. In how many hours would they be able to polish it together ?

45. If 10 labourers can dig a pond in 15 days, how many days will it take 8 labourers to dig the same pond ?

46. If 7 typists typing 7 hours a day (at equal speeds) can type the manuscript of a book in 9 days, then how many days will 3 typists typing 3 hours a day take in typing the manuscript of the book ?

47. A car is travelling at the average speed of 56 km/hr. How much distance will it travel in 45 minutes ?

48. How much time would a bullock-cart moving at an average speed of 4 km/hr take in covering a distance of 18 km ?
49. A cyclist covers a distance of 35 km in 5 hours ? What is his average speed ?
50. 20% of Ramanna's monthly salary is Rs 475. What is Ramanna's monthly salary ?
51. Susheela got 98% marks in Mathematics, 75% marks in Hindi, 62% in English, 86% in Physics and 88% in Chemistry. The total marks allotted to Hindi and English are 150 each. The rest of the subjects carry 100 marks each. Find Susheela's aggregate percentage of marks.
52. As per the rules valid in the year 1986-87, no income tax is levied on the first 18000 rupees of one's annual income. On the next 7000 rupees, 25% income tax is charged. A bank employee's total annual income for 1986-87 was Rs 22788. Calculate the amount of income tax payable by him for the year 1986-87.
53. An alloy of tin and copper consists of 20 parts of tin and 100 parts of copper. Find the percentage of tin in the alloy.
54. Find the C.P. of a table which when sold for Rs 784 brought a profit of 12%.
55. Sulabh purchased a parrot for Rs 25 and a pigeon for Rs 20. He sold the parrot at a profit of 20% but had to sell the pigeon at a loss of 5%. Find his total loss or gain.
56. Abdul sold a television set to Hari at a loss of 10%. Hari spent Rs 300 on its repair. Then he used it for six months. After that he sold it to Shanker for Rs 2100. In this deal, Hari neither gained anything nor lost anything. For how much did Abdul buy the television set ? What was his loss ?
57. Find the simple interest on the sum of Rs 500 for 3 years at 12% per annum. Also, find the amount.
58. What sum of money lent out at 15% per annum simple interest would produce Rs 75 as interest in 2 years ?
59. In how much time will the simple interest on Rs 600 be Rs 300 at 10% per annum ?
60. At what rate per cent per annum will Rs 728 produce Rs 227.50 as simple interest in 5 years ?

61. What money will produce an amount of Rs 174 in 2 years at the rate of 8% per annum simple interest?
62. Mahesh deposited Rs 1000 in a bank. At the end of 2 years, he took out a sum of Rs 200. At the end of 5 years since he deposited the money, he received Rs 528 as interest. Find the annual rate of simple interest.
63. A shop selling knitting machines offers 3% discount on all cash purchases. What cash amount does a customer pay for a knitting machine the price of which is marked as Rs 1746?
64. What price should a tradesman mark on an article which cost him Rs 300 so as to gain 20%, after allowing 5% discount?
65. Find discount per cent if an article marked Rs 3760 is sold for Rs 3008.
66. A shopkeeper marks the prices of his goods 40% above the C.P. After reducing the marked prices of his goods by 10%, he allows a further 2% discount on all cash payments. How much cash a customer will have to pay for an article of which the marked price is Rs 400?
67. $\triangle ABC \cong \triangle PQR$. Which side of triangle PQR is congruent to BC? Which angle of triangle ABC corresponds to angle R?
68. In a triangle ABC, $AB = 5$ cm, $BC = 4$ cm and $CA = 6$ cm. $\triangle PQR \cong \triangle ABC$. What are the lengths of QR and PQ?
69. A square has side 2 cm. Another square has area equal to 4 cm^2 . Are these two squares congruent?
70. Two sides of a rectangle are 3 cm and 4 cm, respectively. Another rectangle also has the same dimensions. What can you say about these rectangles?
71. The radius of a circle is 9 cm. The diameter of another circle is 18 cm. Are these two circles congruent?
72. The area of each of two given rectangles is 12 cm^2 . Must these two rectangles be congruent?
73. Construct a triangle whose sides are 4 cm, 6 cm and 7 cm. Construct another triangle with the same measures. Show by superposing that the two triangles are congruent.
74. Make two triangles with two sides of lengths 3 cm and 5 cm, and included angle equal to 60° . Show that they are congruent.

75. Construct two triangles with two of the angles equal to 30° and 45° . Make the included side 8 cm. Show that these two triangles are congruent.
76. Construct two right-angled triangles with hypotenuse 6 cm and one side 4 cm. Show by superposing that these two triangles are congruent.
77. ABCD is a rectangle whose diagonals meet at the point O. Which of the triangles ABO, BCO and DAO are congruent to each other?
78. ΔPQR is congruent to ΔABC , and ΔABC is congruent to ΔEST . Is ΔPQR congruent to ΔEST ?
79. The sides of a triangle are 9 cm, 12 cm and 15 cm. Is this triangle right-angled?
80. Which of the following are Pythagorean triplets?
(6, 8, 10) , (10, 30, 34) , (14, 48, 50) and (12, 16, 20)
81. Point out the true statements. Also, correct the false ones.
- (a) An altitude of a triangle is a line segment passing through one vertex and perpendicular to the opposite side.
 - (b) The altitudes of a triangle are concurrent.
 - (c) The point where the altitudes of a triangle meet is known as the centroid.
 - (d) The orthocentre of an acute triangle lies in the exterior of the triangle.
 - (e) The orthocentre of a right triangle lies on the triangle itself.
 - (f) An isosceles triangle has equal altitudes.
 - (g) The orthocentre of an equilateral triangle bisects its altitudes.
82. Point out the false statements and correct the same.
- (a) A median of a triangle is a line segment that bisects one of its sides.
 - (b) The medians of a triangle are concurrent.
 - (c) The point of concurrence of the medians is known as the circumcentre.
 - (d) The centroid divides the medians in the ratio 1 : 2.
 - (e) The medians of an equilateral triangle coincide with its altitudes.

- (f) The centroid of an obtuse triangle lies in the exterior of the triangle.
- (g) An isosceles triangle has at least two equal medians.
83. (a) The perpendicular bisectors of the sides of a triangle all meet in a point. What do we call this point?
- (b) The circumcentre of a triangle lies in its interior. What type of a triangle is this?
- (c) Where does the circumcentre of an obtuse triangle lie?
- (d) Where does the circumcentre of a right triangle lie?
- (e) Is the circumcentre of a triangle equidistant from its sides?
- (f) Describe the circumcircle of a triangle.
- (g) What is the relation between the orthocentre, centroid and the circumcentre of an equilateral triangle?
- (h) Describe the incentre of a triangle.
- (i) Is the incentre of a triangle equidistant from its vertices?
- (j) A circle can always be drawn to touch all the three sides of a triangle. What name is given to this circle?
- (k) The bisectors of the angles B and A of a triangle meet at O. What can you say about CO and angle C?

84. Fill in the blanks :

- (a) A quadrilateral has _____ sides, _____ vertices and _____ diagonals.
- (b) The sum of the four angles of a quadrilateral is _____.
- (c) In the quadrilateral ABCD, BD is a _____ and AB is a _____.
- (d) A quadrilateral in which a pair of opposite sides are parallel is known as a _____.
- (e) A quadrilateral in which the opposite sides are parallel is known as a _____.
- (f) In a parallelogram, the opposite sides are _____ and so are the opposite _____.
- (g) Diagonals of a parallelogram _____ each other.
- (h) A rectangle is a _____ in which the angles are right angles.
- (i) Diagonals of a rectangle are _____.
- (j) A parallelogram with all four sides equal is known as a _____.

(k) The diagonals of a rhombus are _____ to each other and also _____ each other.

(l) A square is a rhombus all whose angles are _____.

(m) The diagonals of a square are _____.

85. The base of a parallelogram is 6 cm and the corresponding altitude is 4 cm. Find its area

86. The area of a parallelogram is 84 cm^2 . If its base is 12 cm, then find the corresponding altitude.

87. The area of a parallelogram is 6 cm^2 . If one altitude is 2 cm, then find one side.

88. The base of a triangle is 25 cm and the corresponding altitude is 12 cm. Find its area.

89. The area of a triangle is 46 cm^2 . If one altitude of the triangle is 2 cm, then find one side of the triangle.

90. Find the product of :

(a) $2x$ and $3y^2$

(b) $5xy$ and $15yz$

(c) $7ab^2c$ and $3a^2b^3c$

(d) $24rs^3t^3$ and $10r^2st$

Verify your answers by taking the following values of the variables:

$x = 3$, $y = 2$, $z = 2$, $a = b = 1$, $c = 4$,
 $r = 5$ and $s = t = -2$

91. Multiply by the horizontal method. Verify the solution by the column method.

(a) $xy(3x^2y^2 + 4y)$

(b) $xy^2(x^3 - y^2)$

(c) $(x^2 + y)(6x^3 - 5xy)$

(d) $(p+q)(p^2q + pq^2)$

92. Fill in the blanks :

(a) $(x+y)^2 =$ _____

(b) $(x-y)^2 =$ _____

(c) $(x+y)(x-y) =$ _____

93. Expand using the suitable identities :

$(x+6y)^2$, $(2a - 3b^2)^2$, $(p^2 - 7q^2)^2$

94. Evaluate using the suitable identities :

51^2 , 199^2 , 201^2

95. Write all the possible factors of :

(a) ab

(b) $4sxr$

(c) $4x^2yz$

96. Find the common factors of :

(a) $-3x^2y^3$ and xyz

(b) $12p^2q$ and $42pq^3$

(c) x^2y^2z and xyz^2

97. Factorise :

(a) $4x^2yz + 6xy^2z^3$

(b) $abc - 5ab^2c^3$

(c) $-20abc^3 - 15a^2b^2c^2$

98. Factorise by grouping together suitable terms :

(a) $a^2x + a^2y + b^2x + b^2y$

(b) $ax^2 + by^2 - ay^2 - bx^2$

99. Factorise by making use of suitable identities:

(a) $4x^2 + 12xy + 9y^2$

(b) $a^2 - 4ab + 4b^2$

(c) $64x^2 - 81y^2$

100. Solve and check your solution :

(a) $6x - 13 = 5$

(b) $2x + 5 = 3x - 7$

(c) $4x - 1 = x - 11$

101. Solve and check the solution :

(a) $\frac{2}{3}x + 1 = \frac{4}{3}$

(b) $\frac{5}{6}x - \frac{3}{2} = \frac{7}{2} + x$

102. The following table shows the number of letters delivered to some houses in a particular week in a certain street :

House No.	3	7	10	13	2	1
Number of letters	1	3	14	20	25	29

Draw a bar graph to represent the above data.

CHAPTER ONE

Squares and Square Roots

1.1 Squares

LET US RECALL that

$$2^2 = 2 \times 2 = 4;$$

$$3^2 = 3 \times 3 = 9;$$

$$4^2 = 4 \times 4 = 16.$$

The above statements can also be expressed by saying that the square of 2 is 4, the square of 3 is 9, and the square of 4 is 16. That is, the *square* of a number is that number 'raised to the power 2.'

The adjoining Table 1.1 gives the squares of all natural numbers from 1 to 20.

Table 1.1

The numbers 1,4,9,..., 361, 400 in the table are squares of the numbers 1,2,3,..., 19, 20. Such numbers as 1, 4, 9,..., 361, 400 are called perfect squares.

A natural number is called a *perfect square* or a *square number* if it is the square of some natural number. That is, a natural number n is a perfect square if $n = m^2$, for some natural number m .

Notice that the perfect squares 1, 4, 9, 16,... can be represented geometri-

Number	Square	Number	Square
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

cally by dots forming a square as shown below :



Hence the name 'perfect squares' or 'square numbers'.

The adjoining Table 1.2 gives all numbers between 1 and 400 which are perfect squares.

To find out whether a given number is a perfect square or not, write the number as a product of its prime factors. If the number is a perfect square, you would be able to group the factors in pairs in such a way that both the factors in each pair are equal.

Table 1.2

Perfect Square	Written as	Perfect Square	Written as
1	1^2	121	11^2
4	2^2	144	12^2
9	3^2	169	13^2
16	4^2	196	14^2
25	5^2	225	15^2
36	6^2	256	16^2
49	7^2	289	17^2
64	8^2	324	18^2
81	9^2	361	19^2
100	10^2	400	20^2

Example 1: Is 225 a perfect square?

Solution : $225 = \underline{3 \times 3} \times \underline{5 \times 5}$

We find that the prime factors of 225 can be grouped into pairs as shown and no factor is left over.

$\therefore$ 225 is a perfect square. It is the square of $3 \times 5 = 15$.

Example 2: Is 10584 a perfect square?

Solution : $10584 = \underline{2 \times 2} \times 2 \times \underline{3 \times 3} \times 3 \times \underline{7 \times 7}$

If we try to group the prime factors of 10584 into pairs of equal factors, we find that we are left with two factors

3	225
3	75
5	25
5	

2 and 3 which we cannot pair with equal factors.

$\therefore$ 10584 is *not* a perfect square.

Some Properties of Natural Numbers

There are several interesting properties and patterns about squares of natural numbers.

Let us observe some of these.:

(A) Look up Table 1.2. The squares of the first ten natural numbers end in 0, 1, 4, 5, 6, 9. This suggests that *a number*

ending in 2, 3, 7 or 8 is never a perfect square. Thus, you can immediately say that the numbers 22, 237, 418, etc., are not perfect squares. However, it is not necessary that if a number ends in 0, 1, 4, 5, 6 or 9, then it must be a perfect square. Can you think of some such numbers ?

(B) If a number ends in a zero, you can immediately decide whether it is a perfect square or not. You will notice that *the number of zeros in the end of a perfect square is never odd*. Thus, for example, 45000 is not a perfect square.

(C) Recall that natural numbers are of two types: *even* numbers and *odd* numbers. Even numbers are those natural numbers which are multiples of 2 such as 2, 4, 6, 8,..., and *odd* numbers are those natural numbers which are not multiples of 2 such as 1, 3, 5, 7,....

Now, write down squares of some even numbers and some odd numbers.

$$1^2 = 1$$

$$3^2 = 9$$

$$5^2 = 25$$

$$2^2 = 4$$

$$4^2 = 16$$

$$6^2 = 36 \text{ etc.}$$

You will find that

(1) *Squares of even numbers are always even.*

(2) *Squares of odd numbers are always odd.*

D) Let us observe the following carefully:

$$2^2 = 4 = 3 \cdot 1 + 1, \quad 2^2 = 4 = 4 \cdot 1$$

$$3^2 = 9 = 3 \cdot 3, \quad 3^2 = 9 = 4 \cdot 2 + 1$$

$$4^2 = 16 = 3 \cdot 5 + 1, \quad 4^2 = 16 = 4 \cdot 4$$

$$5^2 = 25 = 3 \cdot 8 + 1, \quad 5^2 = 25 = 4 \cdot 6 + 1$$

$$6^2 = 36 = 3 \cdot 12, \quad 6^2 = 36 = 4 \cdot 9$$

$$7^2 = 49 = 3 \cdot 16 + 1, \quad 7^2 = 49 = 4 \cdot 12 + 1$$

2	10584
2	5292
2	2646
3	1323
3	441
3	147
7	49
7	

The above patterns suggest that

(1) The square of a number (other than 1) is either a multiple of 3 or exceeds multiple of 3 by 1.

(2) The square of a number (other than 1) is either a multiple of 4 or exceeds multiple of 4 by 1.

Can you show that (3) if p^2 is a multiple of 3, then p is also a multiple of 3?

[Hint : If p is not a multiple of 3, then p^2 exceeds a multiple of 3 by 1.]

(E) Can you find two square numbers such that one is double the other? 1 is a square number but $2 \times 1 = 2$ is not a square number. 4 is a square number but

$2 \times 4 = 8$ is not a square number. In fact,

There are no natural numbers p and q such that $p^2 = 2q^2$... (1)

The following argument will convince you :

There are exactly four possibilities:

- (a) p is odd, q is odd.
- (b) p is odd, q is even.
- (c) p is even, q is odd.
- (d) p is even, q is even.

We shall rule out all these possibilities one by one.

(a) In this case, p^2 is odd, but $2q^2$ is even. Therefore, $p^2 \neq 2q^2$.

(b) In this case also, p^2 is odd, but $2q^2$ is even and, therefore, $p^2 \neq 2q^2$.

(c) In this case, p^2 contains 4 as a factor ($\because p$ contains 2 as a factor), whereas $2q^2$ contains 2 as a factor only once (q^2 being odd). Therefore $p^2 \neq 2q^2$.

(d) In this case, p and q both contain 2 as a factor.

Suppose $d = \text{H.C.F. of } p \text{ and } q$.

We can then write

$p = dx$ and $q = dy$, where x and y are natural numbers having no factor (other than 1) in common.

Thus, $p^2 = 2q^2$ gives

$$(dx)^2 = 2(dy)^2$$

That is, $d^2x^2 = 2d^2y^2$.

Cancelling d^2 from both sides, we have $x^2 = 2y^2$.

Since x and y have no factor in common, therefore, at least one of them is odd.

Therefore, this case is reduced to one of the cases (a), (b), (c) and is thus ruled out. Hence, there can be no natural numbers p and q such that $p^2 = 2q^2$.

(F) For every natural number n , we have

$$(n + 1)^2 - n^2 = (n + 1 + n)(n + 1 - n) = (n + 1) + n$$

This will help you to write immediately the difference between two consecutive squares.

$$\begin{aligned} \text{Thus, } 9^2 - 8^2 &= 9 + 8 = 17 \\ 17^2 - 16^2 &= 17 + 16 = 33 \text{ etc.} \end{aligned}$$

(G) Sometimes squares of numbers can be obtained orally by observing a definite pattern such as the following :

$$\begin{aligned} 35^2 &= 3 \times 4 \text{ (hundreds)} + 5^2 = 1225 \\ 45^2 &= 4 \times 5 \text{ (hundreds)} + 5^2 = 2025 \\ 55^2 &= 5 \times 6 \text{ (hundreds)} + 5^2 = 3025 \end{aligned}$$

(H) Can you find three natural numbers m, n, p such that $m^2 + n^2 = p^2$?

As you would recall, such triplets (m, n, p) are called *Pythagorean triplets*. Since $3^2 + 4^2 = 5^2$ and $5^2 + 12^2 = 13^2$, therefore, $(3, 4, 5)$ and $(5, 12, 13)$ are Pythagorean triplets.

You can verify that for any odd number m greater than 1, $\left(m, \frac{m^2 - 1}{2}, \frac{m^2 + 1}{2}\right)$ is a Pythagorean triplet. Similarly, for any number m greater than 1, $(2m, m^2 - 1, m^2 + 1)$ is a Pythagorean triplet.

You can thus obtain lots of Pythagorean triplets for different values of m in the above triplet.

Remark: The following formula gives all primitive Pythagorean triplets (i.e., triplets such that the three numbers contain no common factor) :

$$(2mn)^2 + (m^2 - n^2)^2 = (m^2 + n^2)^2$$

where m and n are relatively prime, $m > n$, and exactly one of m and n is even and the other is odd.

(I) Can you observe a pattern in the following :

$$\begin{aligned} 11^2 &= 1\underline{2}1 \\ 111^2 &= 12\underline{3}21 \\ 1111^2 &= 1234\underline{3}21 \\ 11111^2 &= 1234\underline{5}4321 \\ 111111^2 &= 12345\underline{6}54321 \\ 1111111^2 &= 123456\underline{7}654321 \\ 11111111^2 &= 1234567\underline{8}7654321 \\ 111111111^2 &= 12345678\underline{9}87654321 \end{aligned}$$

The numbers 121, 12321, 1234321, 123454321, and so on have additional interesting properties. The sum of the digits of every such number is a perfect square. For example,

$$\begin{aligned} 1 + 2 + 1 &= 4 = 2^2 \\ 1 + 2 + 3 + 2 + 1 &= 9 = 3^2 \\ 1 + 2 + 3 + 4 + 3 + 2 + 1 &= 16 = 4^2 \\ 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 &= 25 = 5^2 \\ 1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 &= 36 = 6^2 \\ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 6 + 5 + 4 + 3 + 2 + 1 &= 49 = 7^2 \\ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 &= 64 = 8^2 \\ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 &= 81 = 9^2 \end{aligned}$$

EXERCISE 1(a)

(ORAL)

- Which of the following numbers are perfect squares ?
11, 12, 16, 32, 36
- Which of the following perfect squares are squares of even numbers ?
121, 225, 256, 1296, 6561
- Which of the following numbers are perfect squares ?
100, 1000, 330550, 12345600000
- Which of the following numbers are perfect squares ?
22, 121, 343, 373758, 22034087
- Write down the correct number in the box:
 - $23^2 - 22^2 = \boxed{}$
 - $101^2 - 100^2 = \boxed{}$
 - $551^2 - 550^2 = \boxed{}$
- Write down the correct number in each box:
 - $65^2 = \boxed{}$
 - $75^2 = \boxed{}$
- Which of the following triplets are Pythagorean ?
(1, 2, 3), (3, 4, 5), (6, 8, 10), (1, 1, 1), (2, 2, 3)

8. Observe a pattern and find the missing numbers:

$$1^2 + 2^2 + 2^2 = 3^2$$

$$2^2 + 3^2 + 6^2 = 7^2$$

$$3^2 + 4^2 + 12^2 = 13^2$$

$$4^2 + 5^2 + \square^2 = 21^2$$

$$5^2 + \square^2 + 30^2 = 31^2$$

$$6^2 + 7^2 + \square^2 = \square^2$$

9. Observe a pattern in the following and find the missing numbers :

$$121 = \frac{22^2}{1+2+1}$$

$$12321 = \frac{(333)^2}{1+2+3+2+1}$$

$$1234321 =$$

$$123454321 =$$

$$12345654321 =$$

$$1234567654321 =$$

$$123456787654321 =$$

$$12345678987654321 = \frac{(999999999)^2}{1+2+3+4+5+6+7+8+9+8+7+6+5+4+3+2+1}$$

*10. Use your calculator to find the squares of all natural numbers between 80 and 90.

*11. Use your calculator to decide whether the following triplets are Pythagorean :

(a) (15, 36, 39)

(b) (12709, 13500, 18541)

Table 1.3

a	a^2	a	$\sqrt{a}$
1	1	1	1
2	4	4	2
3	9	9	3
4	16	16	4
5	25	25	5
6	36	36	6
7	49	49	7
8	64	64	8
9	81	81	9
10	100	100	10

2 Square Roots

We have seen that the numbers 4, 49, 21 are all perfect squares. That is,

$$4 = 2^2, \quad 49 = 7^2,$$

$$121 = 11^2$$

We say that 2 is the *square root* of 4. Similarly, 7 is the square root of 49, and 11 is the square root of 121.

Thus, x is the square root of a number n if $n = x \times x$. That is, the square root of a given number n is that natural number which when multiplied by itself gives n as the product.

It follows that if a is the square root of b , then b is the square of a .

The square root of a number is indicated by the symbol ' $\sqrt{\quad}$ ' (See Table 1) or ' $\sqrt{\quad}$ '.

For example,

The square root of 4 is written as

$$\sqrt{4} \text{ . Thus, } \sqrt{4} = 2 \text{ .}$$

Similarly, $\sqrt{49} = 7$, $\sqrt{121} = 11$.

(1) The square root of an even perfect square is even.

(2) The square root of an odd perfect square is odd.

1.3 Square Root Through a Pattern

Look at the following geometrical pattern carefully (Fig. 1.1) :

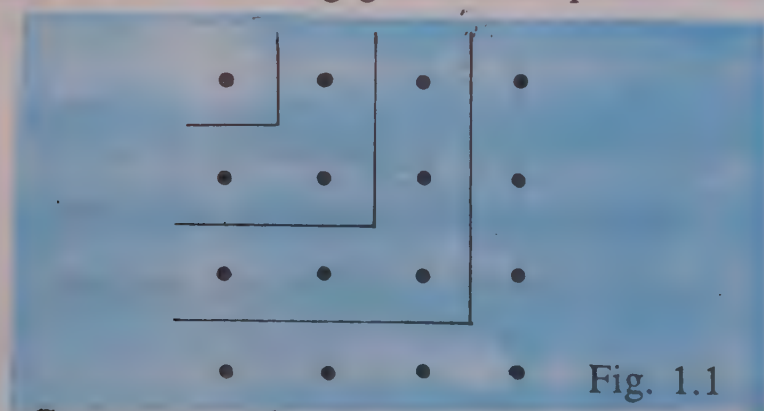


Fig. 1.1

Can you see that

$$\begin{aligned} 1 &= 1^2 \\ 1 + 3 &= 2^2 \\ 1 + 3 + 5 &= 3^2 \\ 1 + 3 + 5 + 7 &= 4^2 \\ 1 + 3 + 5 + 7 + 9 &= 5^2 ? \end{aligned}$$

Thus, if a given number can be written as the sum of first n odd natural numbers, then the number must be the square of the number n . In other words, n will be the square root of given number. We can use this to find the square root of small numbers.

To find the square root of any square number, we subtract from it successive 1, 3, 5, 7, 9, ... The number of times we have to perform subtraction to arrive at zero gives the square root of the given number.

For example, consider 64.

$$\begin{array}{l} \text{Now, } 64 - 1 = 63, \quad 63 - 3 = 60, \quad 60 - 5 = 55, \quad 55 - 7 = 48, \\ \quad \quad 48 - 9 = 39, \quad 39 - 11 = 28, \quad 28 - 13 = 15, \quad 15 - 15 = 0 \end{array}$$

Since, we had to subtract 8 times,

$$\therefore \sqrt{64} = 8$$

EXERCISE 1(b) (ORAL)

Guess and verify the square root of:

- | | | | |
|-------------------|----------------------|--------|---------|
| 1. 25 | 2. 64 | 3. 81 | 4. 100 |
| 5. 169 | 6. 225 | 7. 400 | 8. 4900 |
| 9. 39×39 | 10. 320×320 | | |

4 To Find the Square Root of a Perfect Square by Factors

Look at the following carefully :

$$36 = \underline{2 \times 2} \times \underline{3 \times 3}$$

$$625 = \underline{5 \times 5} \times \underline{5 \times 5}$$

$$1225 = \underline{5 \times 5} \times \underline{7 \times 7}$$

$$\therefore \sqrt{36} = 2 \times 3 = 6$$

$$\therefore \sqrt{625} = 5 \times 5 = 25$$

$$\therefore \sqrt{1225} = 5 \times 7 = 35 \text{ etc}$$

It is clear from the above examples that we have the following rule for finding the square root of a perfect square :

- 1) Write down the prime factors of the given number.
- 2) Since the number is a perfect square, you will be able to make an exact number of pairs of prime factors such that both the factors in each pair are equal. Write one factor from each pair.
- 3) Find the product of such factors.
- 4) The product is the required square root.

Example 1 : Find the square root of 1296.

$$\begin{aligned} \text{Solution : } 1296 &= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3} \\ \therefore \sqrt{1296} &= 2 \times 2 \times 3 \times 3 \\ &= 36 \end{aligned}$$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
	3

Example 2 : Find the smallest number by which the number 1100 must be multiplied (or divided) so that the product (or quotient) becomes a perfect square. Also, in each case find the square root of the perfect square so obtained.

Solution : $1100 = 2 \times 2 \times 5 \times 5 \times 11$

We find that the prime factors 2 and 5 occur in pairs, but there is no prime factor to form a pair with 11. Therefore, we must multiply (or divide) the number by 11 so that it becomes a perfect square.

2	1100
2	550
5	275
5	55
	11

If we multiply the number by 11, the number so obtained

$$\begin{aligned}
 &= 1100 \times 11 = 12100 \\
 &= \underline{2 \times 2} \times \underline{5 \times 5} \times \underline{11 \times 11}, \text{ whose square root} \\
 \sqrt{12100} &= 2 \times 5 \times 11 \\
 &= 110
 \end{aligned}$$

If we divide the number by 11, the number so obtained

$$\begin{aligned}
 &= \frac{1100}{11} = 100 \\
 &= \underline{2 \times 2} \times \underline{5 \times 5}, \text{ whose square root} \\
 \sqrt{100} &= 2 \times 5 \\
 &= 10
 \end{aligned}$$

EXERCISE 1(c)

1. Find the square root of the following by means of factors :

- ~~(a) 16~~ ~~(b) 196~~ ~~(c) 529~~ ~~(d) 400~~
~~(e) 1764~~ ~~(f) 4096~~ ~~(g) 7744~~ ~~(h) 11664~~
~~(i) 4900~~ ~~(j) 47089~~ ~~(k) 1000000~~ ~~(l) 298116~~

2. Find the smallest number by which 180 must be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square so obtained.

3. Find the smallest number by which 9408 must be divided so that it becomes a perfect square. Also, find the square root of the perfect square so obtained.

4. 5929 students are sitting in an auditorium in such a manner that there are as many students in a row as there are rows in the auditorium. How many rows are there in the auditorium?
5. A school collected Rs 2304 as fees from its students. If each student paid as many paise as there were students in the school, how many students were there in the school?
- *6. Use your calculator to find the square roots of numbers given in problem 1.

5. Square Root of a Rational Number

We have talked of square roots of perfect squares. In the same manner, we can talk of the square root of any number which can be written as the square of a rational number. For example, we have

$$\frac{9}{16} = \left(\frac{3}{4}\right)^2$$

$$\frac{121}{625} = \left(\frac{11}{25}\right)^2$$

$$0.01 = (0.1)^2$$

$$0.0144 = (0.12)^2$$

We say that $\frac{3}{4}$ is the square root of $\frac{9}{16}$, $\frac{11}{25}$ is the square root of $\frac{121}{625}$, 0.1 is the square root of 0.01, and 0.12 is the square root of 0.0144.

Thus, x is the square root of a number y if $y = x^2$. That is, *the square root of a rational number m is that rational number which when multiplied by itself gives the given number.*

We write

$$\sqrt{\frac{9}{16}} = \frac{3}{4}, \quad \sqrt{\frac{121}{625}} = \frac{11}{25}, \quad \sqrt{0.01} = 0.1,$$

$$\sqrt{0.0144} = 0.12$$

Remarks : (1) Since $4 = 2^2 = (-2)^2$, therefore, 2 and -2 can be both said to be square roots of 4. However, let us agree that the square root of a number will be taken to be its positive square root. Thus, $\sqrt{4} = +2$, and *not* -2.

(2) The square root of a negative number is not possible. The reason is as follows :

Suppose we consider a negative number $-a$, $a > 0$.

The square root of $-a$ is a number b such that $b \times b = -a$.

Now, if b is positive, $b \times b$ is positive and, therefore, $b \times b$ cannot be equal to the negative number $-a$.

Also, if b is negative, then the product of two negative numbers being positive, $b \times b$ is positive.

It follows, therefore, that there can exist no number b such that $b \times b = -a$, and so the square root of a negative number does not exist. Thus, $\sqrt{-1}$ does not exist.

(3) You must have observed that the square root of square number n is never greater than n . This is not true in the case of rational numbers. For example,

$$\sqrt{\frac{1}{4}} = \frac{1}{2}, \text{ and } \frac{1}{2} > \frac{1}{4}.$$

In fact, if a number is less than 1, then its square root is always greater than the number.

(4) You must be very careful while writing the square root of decimal fraction. Note that $\sqrt{0.4} \neq 0.2$.

You should always check your answer by squaring the number obtained. For example, $0.2 \times 0.2 = 0.04$

Therefore, $\sqrt{0.04} = 0.2$

But $\sqrt{0.4} \neq 0.2$

EXERCISE 1(d) (ORAL)

Guess the square root of :

1. $\frac{1}{4}$

2. $\frac{16}{25}$

3. 0.09

4. 0.0004

5. $\frac{169}{289}$

6. $\frac{121}{10000}$

7. 0.0625 8. 0.0324

6 To Find the Square Root of a Number by the Method of Factors
Examine the following carefully:

$$\begin{aligned}\sqrt{9 \times 100} &= \sqrt{900} = \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5} \\ &= 2 \times 3 \times 5 = 30 = 3 \times 10 = \sqrt{9} \times \sqrt{100}.\end{aligned}$$

$$\sqrt{\frac{4}{9}} = \sqrt{\frac{2 \times 2}{3 \times 3}} = \sqrt{\frac{2}{3} \times \frac{2}{3}} = \frac{2}{3} = \frac{\sqrt{4}}{\sqrt{9}}$$

From the above examples, it is clear that if a and b are perfect squares, then

$$\begin{aligned}\sqrt{a \times b} &= \sqrt{a} \times \sqrt{b} \\ \sqrt{\frac{a}{b}} &= \frac{\sqrt{a}}{\sqrt{b}}\end{aligned}$$

Example 1 : Find the square root of 1082.41.

Solution : $1082.41 = .01 \times 108241$
 $= .1 \times .1 \times 7 \times 7 \times 47 \times 47$

Now, $\sqrt{1082.41} = \sqrt{.01} \times \sqrt{108241}$

$\therefore \sqrt{1082.41} = .1 \times 7 \times 47$
 $= 32.9$

7	108241
7	15463
47	2209
	47

Example 2: Find the square root of $\frac{256}{441}$.

Solution: $\sqrt{\frac{256}{441}} = \frac{\sqrt{256}}{\sqrt{441}}$

Now, $\sqrt{256} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$
 $= 2 \times 2 \times 2 \times 2 = 16$

$\sqrt{441} = \sqrt{3 \times 3 \times 7 \times 7} = 3 \times 7 = 21$

$\therefore \sqrt{\frac{256}{441}} = \frac{\sqrt{256}}{\sqrt{441}} = \frac{16}{21}$

2	256	3	441
2	128	3	147
2	64	7	49
2	32		7
2	16		
2	8		
2	4		
	2		

Example 3 : Find the square root of $38\frac{11}{25}$.

$$\begin{aligned}\text{Solution: } 38\frac{11}{25} &= \frac{961}{25} \\ &= \frac{31 \times 31}{5 \times 5}\end{aligned}$$

$$\begin{aligned}\therefore \sqrt{38\frac{11}{25}} &= \sqrt{\frac{31 \times 31}{5 \times 5}} \\ &= \frac{\sqrt{31 \times 31}}{\sqrt{5 \times 5}} = \frac{31}{5} = 6\frac{1}{5}\end{aligned}$$

It is clear that we have the following rule for finding the square root of a fraction :

- (1) If the given fraction is a mixed fraction, then first convert it into an improper fraction.
- (2) Find the square root of the numerator and the denominator separately.
- (3) The fraction whose numerator is the square root of the numerator of the given fraction, and denominator is the square root of the denominator of the given fraction, is the required square root.

EXERCISE 1(e)

Find the square root of :

1. $\frac{625}{1296}$
2. $4\frac{29}{49}$
3. $2\frac{137}{196}$
4. $23\frac{26}{121}$
5. $52\frac{857}{2116}$
6. $75\frac{46}{49}$
7. 5.774409
8. .00053361
9. 804609

10. The area of a square field is $101\frac{1}{400}$ square metres. Find the length of one side of the field.

*11. Use your calculator to find the square roots of all numbers given in problems 1 to 9 above. Also, verify your answer in each case.

7 To Find the Square Roots of Perfect Squares by the Method of Long Division

We have seen how to find the square root of a number by means of its factors. However, sometimes it is not always easy to write the factors of a number. In such cases, we use the method of long division to find the square root. But before we give examples to establish and illustrate the algorithm (method), let us discuss the pattern of the number of digits in a number and the number of digits in its square root.

Squares of numbers from 1 to 9 have at most two digits; for instance, squares of 1, 2, 3 are 1, 4, 9 respectively and squares of 4, 5, ..., 9 are 16, 25, ..., 81 respectively.

Since the square of 10 is 100 and that of 99 is 9801, therefore, the squares of numbers from 10 to 99 have either 3 digits or 4 digits. Similarly, the squares of numbers from 100 to 999 have either five digits or six digits.

Therefore, it is clear that if there are 2 or less than 2 digits in a perfect square, then the square root of that number will have only one digit. If a number is a perfect square and consists of either 3 or 4 digits, then its square root consists of 2 digits. If a number is a perfect square and consists of 5 or 6 digits, then the square root of the

number consists of 3 digits. These facts are summarized in the adjoining Table 1.4.

Table 1.4

<i>No. of digits in a perfect square number</i>	<i>No. of digits in the square root of that number</i>
1 or 2	1
3 or 4	2
5 or 6	3

From the above Table 1.4, we observe that (1) if the number of digits in a square number is even, then the number of digits in its square root is half of the number of digits in the number, and (2) if the number of digits in a square number is odd, then the number of digits in the square root is half of the number obtained by adding 1 to the number of digits in the given number.

If you want to find the number of digits in the square root of a number, place a bar over every pair of digits starting with the units digit and count the number of these bars. The number of bars will be the number of digits in the square root.

For example,

Square root of $\overline{2} \overline{56}$ will have two digits (2 will be called the first period and 56 will be called the second period).

Square root of $\overline{12} \overline{25}$ will have two digits (12 is the first period, 25 is the second period). Square root of $\overline{1} \overline{46} \overline{41}$ will have three digits (1 is the first period, 46 is the second period, 41 is the third period).

Square root of $\overline{78} \overline{32} \overline{25}$ will have three digits (78 is the first period, 32 is the second period, 25 is the third period).

Example 1: Find the square root of 529.

Solution : Observe that 529 has three digits. Therefore, its square root will have two digits.

If we consider the perfect squares 10^2 , 20^2 , 30^2 etc., we find that

$$20^2 = 400 < 529 < 900 = 30^2.$$

It is clear that $\sqrt{529}$ lies between 20 and 30 (larger the number, larger the square root). That is, $\sqrt{529}$ is one of the following numbers:

$$21, 22, \dots, 29.$$

$\sqrt{529}$ is thus a two-digit number with 2 in the ten's place.

Let us write $\sqrt{529} = 20 + h$, so that h is the digit in the units place of $\sqrt{529}$.

We have

$$529 = (20 + h)^2 = 400 + 40h + h^2$$

$$\text{or, } 529 - 400 = h(2 \times 20 + h)$$

This tells us how we can find h .

We must have

$$129 + (2 \times 20 + h) = h$$

Since $12 + 4 = 16$, we guess that $h = 3$, and it is true.

$$\text{Thus, } 129 - 3 \times 43 = 0$$

To summarize,

(i) After first step, remainder

$$129 = 529 - 20^2$$

(ii) After second step, remainder

$$0 = 529 - 23^2$$

We have found the square root 23 in two steps, 20 and then 23. In short, we may show the working as follows :

	23
2	$\overline{5\ 29}$
	4
43	$\overline{129}$
	129
	$\overline{0}$

The above method is quite general. We shall take another example and then state the algorithm.*

Example 2 : Find the square root of 6561.

Solution :

	81
8	$\overline{65\ 61}$
	64
161	$\overline{161}$
	161
	$\overline{0}$

$$\therefore \sqrt{6561} = 81$$

Square Root Algorithm

- 1) Place bars over every pair of digits starting with the units digit. The number of bars indicates the number of digits in the square root. (Observe that there may be only one digit in the first period.)
- 2) Think of the largest square less than or equal to the first period. Take the square root of this number as divisor and as quotient. Put the quotient above the period. Subtract the product and bring down the next period to the right of the remainder.

	$\overline{65\ 61}$
	8
8	$\overline{65\ 61}$
	64
	$\overline{161}$

Algorithm is another way of expressing the computational work spelled out in a definite sequence of steps.

- (3) Double the quotient as it appears and enter it with a blank on the right for the next digit, as the next possible divisor.
- (4) Guess a possible digit to fill the blank and also to become the new digit in the quotient. Enter this digit in both places. Multiply the new digit in the quotient and the new divisor. Subtract and bring down the next period (if any).
- (5) Repeat Steps 3 and 4, till the last period has been dealt with.

$$\begin{array}{r|l}
 & 8 \\
 8 & \overline{65\ 61} \\
 & 64 \\
 \hline
 16- & 161
 \end{array}$$

$$\begin{array}{r|l}
 & 81 \\
 8 & \overline{65\ 61} \\
 & 64 \\
 \hline
 161 & 161 \\
 & 161 \\
 \hline
 & 0
 \end{array}$$

Remark : In the example above, there were only two periods and so the algorithm stopped after Step 4. In the next few examples, we shall only give the solution. You are advised to refer to the statement of the algorithm above, when in doubt.

Example 3 : Find the square root of 531441.

Solution :

$$\begin{array}{r|l}
 & 729 \\
 7 & \overline{53\ 14\ 41} \\
 & 49 \\
 \hline
 142 & 414 \\
 & 284 \\
 \hline
 1449 & 13041 \\
 & 13041 \\
 \hline
 & 0
 \end{array}$$

$$\therefore \sqrt{531441} = 729$$

Remark : In the last step, to guess 9, we had to try $1304 \div 144$ (leaving one digit) or $130 \div 14$ (leaving two digits).

Example 4 : Find the square root of 363609.

Solution :

	603
6	<u>36</u> <u>36</u> <u>09</u>
	36
120	<u>36</u>
	00
1203	<u>3609</u>
	3609
	0

$$\therefore \sqrt{363609} = 603$$

Remark : At the second stage, to guess 0, we had to try $3 \div 12$ (leaving one digit in each number).

Example 5 : Find the square root of 4401604.

Solution :

	2098
2	<u>4</u> <u>40</u> <u>16</u> <u>04</u>
	4
40	<u>40</u>
	00
409	<u>4016</u>
	3681
4188	<u>33504</u>
	33504
	0

$$\therefore \sqrt{4401604} = 2098$$

Remark : At the second stage if we divide 4 by 4, we get 1. But if we put 1 to the right of the divisor 4, it becomes 41 which exceeds the dividend 40. Therefore, instead of 1, we place a zero there. In the next step, if we divide 401 by 40, the quotient is 10. But the next digit can be atmost 9. We, therefore, take 9 as the digit to the right of 40.

Example 6 : Find the least number which must be subtracted from 194491 to make it a perfect square.

Solution : If we work out the process of finding the square root by the division method, we find that if 10 be subtracted from the given number, the square root of the remainder will be 441. It follows, therefore, that in that case the given number will be a perfect square. Hence, 10 is the required number.

	441
4	$\overline{19\ 44}$
	16
84	$\overline{344}$
	336
881	$\overline{89}$
	881
	1

Example 7 : Find the least number which must be added to 306452 to make a perfect square.

Solution :

	553
5	$\overline{30\ 64\ 52}$
	25
105	$\overline{564}$
	525
1103	$\overline{3952}$
	3309

	554
5	$\overline{30\ 64\ 52}$
	25
105	$\overline{564}$
	525
1104	$\overline{3952}$
	4416

From the above working, it is clear that the given number is greater than $(553)^2$ but less than $(554)^2$. If in the given number, we add $(4416 - 3952 = 464)$, then the sum will be a perfect square.

Hence, the required least number is 464.

EXERCISE 1(f)

- Find the number of digits in the square roots of the following numbers :

81 , 121 , 9801 , 99856 , 998001 , 657666025

Find the square root of :

- | | | | |
|----------------|----------------|----------------|--------------|
| 2. 2304 | 3. 4489 | 4. 3481 | 5. 9801 |
| 6. 44100 | 7. 27225 | 8. 54756 | 9. 49284 |
| 10. 99856 | 11. 390625 | 12. 119025 | 13. 193600 |
| 14. 1234321 | 15. 4937284 | 16. 21224449 | 17. 82264900 |
| 18. 64432729 | 19. 62504836 | 20. 3915380329 | |
| 21. 3226694416 | 22. 6407522209 | | |

23. Find the least number which must be subtracted from 2361 to make it a perfect square.

24. Find the least number which must be added to 4931 to make it a perfect square.

25. Find the least number which must be subtracted from 18265 to make it a perfect square.

26. Find the least number which must be added to 4515600 to make it a perfect square.

27. Find the least number of four digits which is a perfect square.

28. Find the greatest number of six digits which is a perfect square.

29. A General arranges his soldiers in rows to form a perfect square. He finds that in doing so, 60 soldiers are left out. If the total number of soldiers be 8160, find the number of soldiers in each row.

*30. Use your calculator to find the square roots of all numbers in problems 1 to 22 above. Also, verify the answer.

3 To Find the Square Roots of Fractions by the Method of Long Division
e shall illustrate the method by means of an example.

Example 1 : Find the square root of $21\frac{2797}{3364}$.

Solution : $21\frac{2797}{3364} = \frac{73441}{3364}$

$$\begin{array}{r|l} & 271 \\ \hline 2 & \overline{73441} \\ & 4 \\ \hline 47 & \overline{334} \\ & 329 \\ \hline 541 & \overline{541} \\ & 541 \\ \hline & 0 \end{array}$$

$$\begin{array}{r|l} & 58 \\ \hline 5 & \overline{3364} \\ & 25 \\ \hline 108 & \overline{864} \\ & 864 \\ \hline & 0 \end{array}$$

$$\begin{aligned} \text{Now, } \sqrt{21\frac{2797}{3364}} &= \sqrt{\frac{73441}{3364}} = \frac{\sqrt{73441}}{\sqrt{3364}} \\ &= \frac{271}{58} = 4\frac{39}{58} \end{aligned}$$

Let us now discuss the method of finding the square root of decimal fraction by division. You will find that the method is just an extension of the usual square root algorithm.

We have :

$$\begin{aligned} (.4)^2 &= .16 & , & \quad \therefore \sqrt{.16} = .4 \\ (.41)^2 &= .1681 & , & \quad \therefore \sqrt{.1681} = .41 \\ (4.1)^2 &= 16.81 & , & \quad \therefore \sqrt{16.81} = 4.1 \end{aligned}$$

From the above examples, it is clear that the square of a decimal fraction consists of twice as many decimal places as given in the number. It follows therefore, that the number of decimal places in the square root of a given decimal fraction is half of the number of decimal places in the given number. Therefore the tenth's digit of the square root comes from the first two decimal places to the right, hundredth's from the third and fourth places and so on.

In view of the above observations, we have the following algorithm to find the square root of a decimal fraction say 37.0881 :

(1) Place bars on the integral part as usual.

$\overline{37.0881}$

1) Place bars on the decimal part on every pair of digits beginning with the first decimal place.

2) Start finding the square root as usual.

$$\overline{37}.\overline{08}\overline{81}$$

	6
6	$\overline{37}.\overline{08}\overline{81}$
	36
12	108

3) A decimal point is placed in the square root as soon as the integral part is exhausted.

	6.09
6	$\overline{37}.\overline{08}\overline{81}$
	36
120	108
	0
1209	10881
	10881
	0

4) Complete the algorithm as usual.

Example 2 : Find the square root of .00053361.

Solution :

	.0231
2	$\overline{.00}\overline{05}\overline{33}\overline{61}$
	4
43	133
	129
461	461
	461
	0

∴ the required square root is .0231.

Remark : Note that in Example 2, there is no integral part before the decimal point. Therefore, in the square root also there will be no number before the decimal point. So place the decimal point before starting the solution. Since there are two zeros immediately after the decimal point, so place a 0 in the square root and proceed in the usual manner to complete the solution.

EXERCISE 1(g)

Find the square root of :

1. $\frac{361}{625}$

2. $5\frac{19}{25}$

3. $34\frac{15}{49}$

4. $84\frac{37}{121}$

5. $23\frac{394}{729}$

6. $21\frac{51}{169}$

7. $10\frac{151}{225}$

8. $35\frac{85}{1444}$

9. $332\frac{61}{169}$

10. 7.29

11. 16.81

12. 9.3025

13. 84.8241

14. 150.0625

15. 477.4225

16. 225.6004

17. .00008281

18. .053361

19. A decimal fraction is multiplied by itself. If the product is 251953.8025, find the fraction.

20. What is that fraction which when multiplied by itself gives 227.798649 ?

21. The area of a square playground is 256.6404 square metres. Find the length of one side of the playground.

*22. Use your calculator to determine the square roots of all numbers given in problems 1 to 18 above. Also, verify your answers by finding the squares of the square roots obtained.

1.9 The Square Root of 2

We have defined square roots of numbers, which can be written as square numbers. What about the square root of 2 ? Can we write 2 as the square number ? In other words, is there a number whose square is 2 ?

Let us examine the natural numbers.

$$1^2 = 1 < 2, \quad 2^2 = 4 > 2.$$

Larger numbers will have still larger squares. It follows that there is no natural number whose square is 2.

Is there a rational number whose square is 2? If $\frac{p}{q}$ is a rational number whose square is 2, then we shall have $\left(\frac{p}{q}\right)^2 = 2$, that is,

$$\frac{p^2}{q^2} = 2, \text{ or } p^2 = 2q^2.$$

But you have seen earlier that there exist no p and q such that $p^2 = 2q^2$. Therefore, there is no rational number whose square is 2.

Is there a negative number $-m$ ($m > 0$) whose square is 2?

Now, $(-m)^2 = 2$ means $m^2 = 2$ which has been ruled out. It follows, therefore, that there is no rational number whose square is 2.

But, in practical situations, we do come across lengths which are represented by a number whose square is 2. For example, consider a square field ABCD (Fig. 1.2) one of whose sides is equal to 1 metre.

What is the length of its diagonal?

We have

$$\begin{aligned} a^2 &= AB^2 + BC^2 \\ &= 1 + 1 = 2 \end{aligned}$$

That is, the length of the diagonal AC (in metres) is a number whose square is 2.

Similarly, suppose we are given a square field whose area is 2 m^2 (Fig. 1.3). What is the length x (in metres) of a side of the field? Obviously, $x^2 = 2$. That is, the length (in metres) of a side is equal to a number whose square is 2.

Thus, if we define the square root of 2 as a 'number' whose square is 2, then we know that this 'number' is not a rational number. Such numbers are called irrational numbers. Hence, $\sqrt{2}$ is irrational. But it is possible to find a rational number whose square is 'nearly' equal to 2. Let us see how we do it.

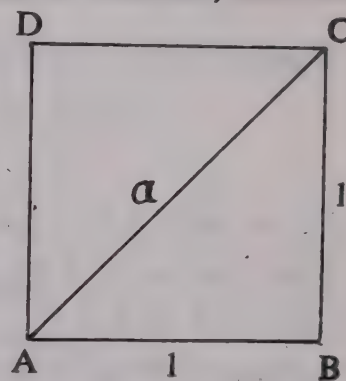


Fig. 1.2

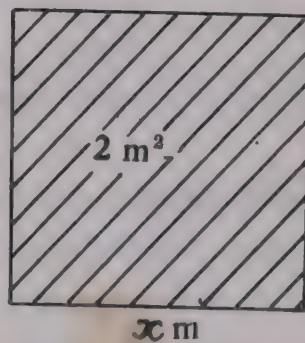


Fig. 1.3

Obviously,

$$1^2 < 2 < 2^2$$

$$\therefore 1 < \sqrt{2} < 2$$

That is, $\sqrt{2}$ is a number lying between 1 and 2.

Now, 1.1, 1.2, ..., 1.9 are all rational numbers between 1 and 2. Let us calculate their squares:

(If you have access to a calculator, use your calculator to write these squares)

Table 1.5

x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
x^2	1	1.21	1.44	1.69	1.96	2.25	2.56	2.89	3.24	3.61

You will find that the square of none of these numbers is 2. However,

$$(1.4)^2 = 1.96 < 2 < 2.25 = (1.5)^2$$

$$\therefore 1.4 < \sqrt{2} < 1.5$$

Now, which are the rational numbers between 1.4 and 1.5 ?

The numbers 1.41, 1.42, 1.43, ..., 1.49 are all rational numbers which lie between 1.4 and 1.5.

Again, you calculate squares of all these numbers. (You may use your calculator if you have one.) You will find that

$$(1.41)^2 < 2 < (1.42)^2$$

$$\text{Thus, } 1.41 < \sqrt{2} < 1.42$$

Another similar step would give that

$$1.414 < \sqrt{2} < 1.415$$

You will find that if you continue in this manner, the process will never come to an end. That is, we can keep on getting a rational number which is nearer and nearer to $\sqrt{2}$. Hence, although $\sqrt{2}$ is not a rational number, it is possible to obtain its approximate value as a rational number.

1.10 To Find the Approximate Values of Square Roots by the Method of Long Division

The method of long division can be used to obtain square roots correct to certain decimal places. This can be done by adding a suitable number of zeros at the extreme right of the decimal part of the number. The method is best illustrated by means of examples.

Example 1 : Find the square root of 3 correct to three places of decimal.

Solution : Since we need the answer correct to three places of decimal, we shall first find the square root upto four places of decimal. For this purpose, we must add 8 (i.e. four pairs of) zeros to the right of the decimal point.

	1.7320
1	<u>3.00 00 00 00</u>
	1
27	<u>200</u>
	189
343	<u>1100</u>
	1029
3462	<u>7100</u>
	6924
34640	<u>17600</u>

$\therefore \sqrt{3} = 1.7320$ up to four places of decimal.
 $= 1.732$, correct to three places of decimal.

Thus, the required square root is 1.732.

Remark: Since the digit in the fourth place (0) is less than 5, therefore, we have not changed it. If the digit in the fourth place were 5 or more than 5, then for writing the answer correct to three decimal places, we would have added 1 to the digit in the third place.

Example 2 : Find the square root of 2.8 upto three places of decimal.

Solution : Since we have to find the square root upto three places of decimal, therefore, we should have eight digits to the right of the decimal point in the given number. So write seven zeros to the right of 8.

$\therefore \sqrt{2.8}$ calculated upto three places of decimal is 1.673.

	1. 6733
1	<u>2. 80 00 00 00</u>
	1
26	<u>180</u>
	156
327	<u>2400</u>
	2289
3343	<u>11100</u>
	10029
33463	<u>107100</u>
	100389
	<u>6711</u>

cond Method

$$10 \frac{2}{3} = \frac{32}{3} = \frac{32 \times 3}{3 \times 3} = \frac{96}{9}$$

$\therefore \sqrt{10 \frac{2}{3}} = \frac{\sqrt{96}}{\sqrt{9}} = \frac{\sqrt{96}}{3}$. Let us find $\sqrt{96}$ by division method.

	9.7979....
9	<u>96.00 00 00 00</u> 81
187	<u>1500</u> 1309
1949	<u>19100</u> 17541
19587	<u>155900</u> 137109
195949	<u>1879100</u> 1763541

$$\therefore \sqrt{96} = 9.7979...$$

Hence, $\sqrt{10 \frac{2}{3}} = \frac{\sqrt{96}}{3} = \frac{9.7979}{3} = 3.2659 = 3.266$, correct to three decimal places.

Remark: If you write $\sqrt{\frac{32}{3}}$ as $\frac{\sqrt{32}}{\sqrt{3}}$ and try to calculate $\sqrt{32}$ and $\sqrt{3}$, the process will prove to be much more difficult.

EXERCISE 1(h)

Find the square root correct to three places of decimal :

1. 1.7

2. 23.1

3. 5

4. 237.615

11.11.11

5. .9 6. $\sqrt{20}$ 7. $\sqrt{.1}$ 8. .016
9. .00064 10. $\sqrt{7}$ 11. $\sqrt{\frac{7}{8}}$ 12. $\sqrt{66}$
13. 15.3215 14. 2.5 15. $\sqrt{\frac{5}{12}}$ 16. $2\sqrt{\frac{1}{12}}$
17. $\sqrt{287\frac{5}{8}}$ 18. $\sqrt{367\frac{2}{7}}$
19. Find the square root of 12.0068 correct to four decimal places.
20. Find the square root of 11 correct to five decimal places.
- *21. Use your calculator to write square roots of all numbers given in problems 1 to 20 and verify your answers.

1.11 The Use of Square Root Tables to Determine Approximate Values of Square Roots

We have seen that the square root of 2 is not a rational number. In fact, square roots of many numbers are not rational numbers. If you look at Table 1.2, you will see that from 1 to 400, there are only 20 natural numbers which are perfect squares, namely, 1, 4, 9, ..., 361, 400. The square roots of the remaining 380 natural numbers are irrational. In such cases, we try to find a rational number whose square is nearly equal to the given number. In many practical problems, we need square roots of numbers for use in the solution. Finding approximate values of square roots by the method of long division or by approximation as in the case of $\sqrt{2}$ in Section 1.9, is very time-consuming and cumbersome. For this reason, tables have been prepared which list these approximate values of square roots of different numbers. Square roots of most numbers can be written down with the help of these tables. Let us learn the use of these square root tables. Since we shall be getting approximate values only, we shall use the symbol ' $\approx$ ' to indicate approximate same.

Look at Table 1.6. It gives values of square roots of all natural numbers from 1 to 99.

Example 1 : Find the value of $\sqrt{5}$ from the table.

Solution : Look at the row containing 5. The entry in the column of $\sqrt{x}$ shows 2.236. Therefore, for $x = 5$,

$$\sqrt{x} = \sqrt{5} \approx 2.236$$

$$\therefore \sqrt{5} \approx 2.236$$

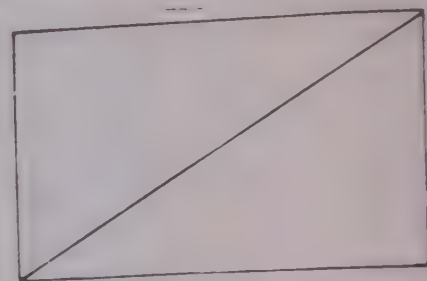
Table 1.6

x	$\sqrt{x}$	x	$\sqrt{x}$
1	1.000	50	7.071
2	1.414	51	7.141
3	1.732	52	7.211
4	2.000	53	7.280
5	2.236	54	7.348
6	2.449	55	7.416
7	2.646	56	7.483
8	2.828	57	7.550
9	3.000	58	7.616
10	3.162	59	7.681
11	3.317	60	7.746
12	3.464	61	7.810
13	3.606	62	7.874
14	3.742	63	7.937
15	3.873	64	8.000
16	4.000	65	8.062
17	4.123	66	8.124
18	4.243	67	8.185
19	4.359	68	8.246
20	4.472	69	8.307
21	4.583	70	8.367
22	4.690	71	8.426
23	4.796	72	8.485
24	4.899	73	8.544
25	5.000	74	8.602
26	5.099	75	8.660
27	5.196	76	8.718
28	5.292	77	8.775
29	5.385	78	8.832
30	5.477	79	8.888
31	5.568	80	8.944
32	5.657	81	9.000
33	5.745	82	9.055
34	5.831	83	9.110
35	5.916	84	9.165
36	6.000	85	9.220
37	6.083	86	9.274
38	6.164	87	9.327
39	6.245	88	9.381
40	6.325	89	9.434
41	6.403	90	9.487
42	6.481	91	9.539
43	6.557	92	9.592
44	6.633	93	9.644
45	6.708	94	9.695
46	6.782	95	9.747
47	6.856	96	9.798
48	6.928	97	9.849
49	7.000	98	9.899
		99	9.950

Example 2 : Find the approximate length of the diagonal of a rectangular piece of land whose sides are 8m and 5m (Fig. 1.4).

Solution : The required length of the diagonal

$$\begin{aligned}
 &= \sqrt{8^2 + 5^2} \text{ m} \\
 &= \sqrt{64 + 25} \text{ m} \\
 &= \sqrt{89} \text{ m} \\
 &\approx 9.434 \text{ m [See Table 1.6]}
 \end{aligned}$$



8 m
Fig. 1.4

It is clear that just by looking at the table, you can easily write the value of $\sqrt{x}$ for any natural number x from 1 to 100. However, we can find the value of $\sqrt{x}$ for any number x between 1 and 100 with the help of this table. We shall illustrate this point by means of an example.

Example 3 : Find the value of $\sqrt{13.32}$.

Solution : We know that

$$\begin{aligned}
 13 &< 13.32 < 14. \\
 \therefore \sqrt{13} &< \sqrt{13.32} < \sqrt{14}.
 \end{aligned}$$

From the square root table,

$$\sqrt{14} = 3.742$$

$$\text{and } \sqrt{13} = 3.606$$

$$\text{Difference} = .136$$

$$\text{Difference for } (14 - 13) \text{ i.e. } 1 = .136$$

$$\therefore \text{Difference for } .32 = .136 \times .32 = .044 \text{ (upto 3 places of decimal)}$$

$$\sqrt{13} = 3.606$$

$$\text{Difference} = .044$$

$$\therefore \sqrt{13.32} = 3.606 + .044 = 3.650$$

Remark : Strictly speaking, all the above values are approximate, and, therefore we should have used ' $\approx$ ' instead of '=' everywhere. However, for the sake of convenience, there is no harm in using '=', if we remember that the values are only approximate.

In fact, the method of the above example can be used to obtain square roots of all numbers from 1 to 10000.

Example 4 : Find the value of $\sqrt{1332}$.

Solution : We can write $1332 = 13.32 \times 100$.

$$\begin{aligned}\therefore \sqrt{1332} &= \sqrt{13.32 \times 10^2} = 10 \sqrt{13.32} \\ &= 10 (3.650) \\ &= 36.5\end{aligned}$$

Remark : Notice that $\sqrt{1332}$ can also be evaluated by observing that

$$1332 = 2^2 \times 3^2 \times 37.$$

$\sqrt{1332} = 2 \times 3 \times \sqrt{37} = 6 \sqrt{37}$. In fact, you can find square roots of several numbers by an application of the rules

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, which hold for all positive numbers a and b but may not necessarily be perfect squares.

Example 5 : Find the value of $\sqrt{31428}$.

$$\begin{aligned}\text{Solution : } \sqrt{31428} &= \sqrt{2^2 \times 9^2 \times 97} \\ &= 2 \times 9 \times \sqrt{97}\end{aligned}$$

Now, write the value of $\sqrt{97}$ from the table to complete the answer.

Example 6 : Find the value of $\sqrt{\frac{37}{64}}$.

$$\begin{aligned}\text{Solution : } \sqrt{\frac{37}{64}} &= \frac{\sqrt{37}}{\sqrt{64}} = \frac{1}{8} \times \sqrt{37} \\ &= \frac{1}{8} \times (6.083) = .7603\end{aligned}$$

EXERCISE 1(i)

Using Table 1.6, find the square roots of :

1. $\sqrt{7}$

2. $\sqrt{15}$

3. $\sqrt{82}$

4. $\sqrt{\frac{3}{25}}$

5. 540

6. 8700

7. 1312

8. 13.21

9. 55.62 10. 71.79 11. 4192 12. 4955
13. 25725 14. $\frac{101}{169}$

15. The area of a square plot of land is 325 square metres. Find the approximate length of one side of the plot.
16. Find the approximate length of the side of a square whose area is equal to the area of a rectangle, with sides 16 metres and 15 metres.
- *17. Use your calculator to solve problems 1 to 14. Also, verify your answers.

1.12 Extension of the Number System to Real Numbers

We have seen that the process of taking square roots gives us numbers which are not rationals. However, there are lengths which are represented by $\sqrt{2}$, $\sqrt{3}$, $\sqrt{7}$, etc.

In Class VII, you learnt that between any two rational numbers, there is another rational number. Therefore, when the rationals are represented on a number line, between any two points, we can insert a point representing a rational number. You can go on doing this indefinitely. However close you take the two points, the process will not come to an end. Intuitively, you are led to believe that the whole line is composed of rational points. Although rationals exist on the line densely, they do not fill out the line completely. There are many points (more than the rationals in fact) on the number line which cannot be represented by rational numbers. This can be seen as follows :

Suppose on the number line, the point O represents zero and A represents integer 1, so that the length $OA = 1$ (See Fig. 1.5).

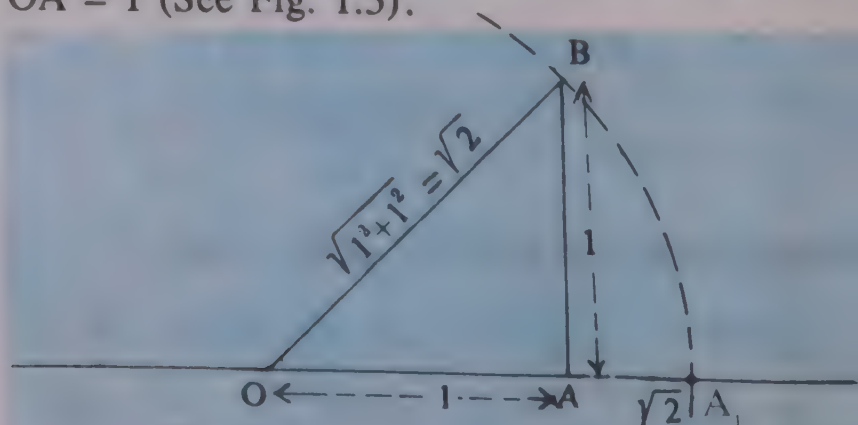


Fig. 1.5

Now, draw a right-angled triangle OAB so that $AB = OA = 1$. By Pythagoras theorem, we have $OB^2 = OA^2 + AB^2 = 1^2 + 1^2 = 1 + 1 = 2$

Therefore, $OB = \sqrt{2}$.

If, now, we draw a circle with centre as O and radius OB , then the circle will cut the number line at A_1 such that $OA_1 = \sqrt{2}$. Thus, there is a point A_1 on the number line which represents $\sqrt{2}$.

But we have seen that $\sqrt{2}$ is not a rational number. We are convinced again that there are numbers which are not rationals.

All points on the number line which do not represent rational numbers represent *irrational* numbers. The system of all rational and irrational numbers is called the system of real numbers and is denoted by $\mathbb{R}$.

A construction similar to the one given in Fig. 1.5 would convince you that there are many other points on the line which are not represented by a rational number, such as $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, etc.

All these numbers arise as the square root of a natural number which is not a perfect square. We repeat the construction of Fig. 1.5 as follows :

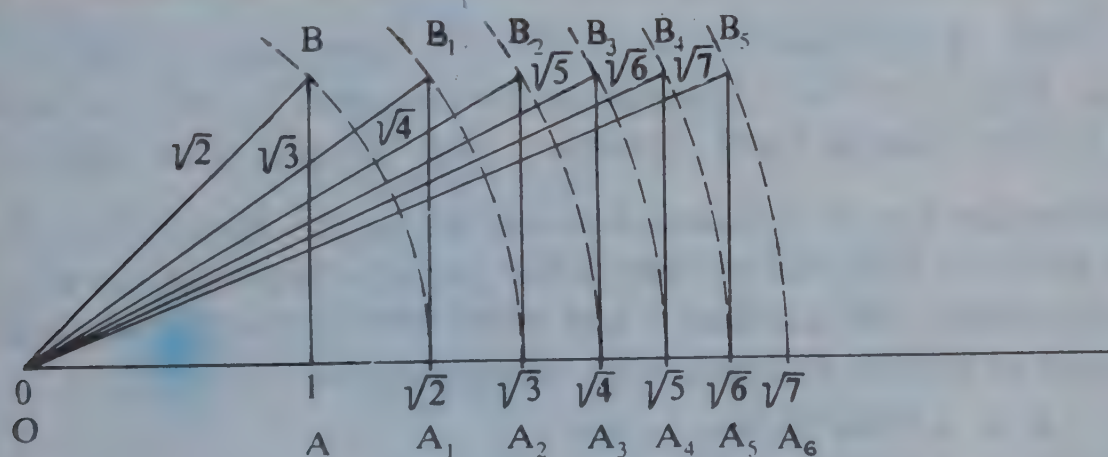


Fig. 1.6

We have $OA_1 = \sqrt{2}$. Complete the right angled triangle OA_1B_1 with $A_1B_1 = 1$ (Fig. 1.6). Then, by Pythagoras theorem,

$$\begin{aligned} OB_1^2 &= OA_1^2 + A_1B_1^2 \\ &= (\sqrt{2})^2 + 1^2 = 2 + 1 = 3 \end{aligned}$$

Therefore, $OB_1 = \sqrt{3}$.

The circle with O as centre and OB_1 as radius cuts the number line at A_2 such that $OA_2 = \sqrt{3}$. So, A_2 represents $\sqrt{3}$ which is not a rational number.

We may now go on repeating the construction with OA_2 to get A_3 such that $OA_3 = \sqrt{4} = 2$ which is not an irrational number, then with OA_3 to get $A_4 = \sqrt{5}$ so on.

In Class VII, you learnt that every rational number can be expressed as a terminating or as a non-terminating repeating decimal, and that every terminating or every repeating decimal represents a rational number. In view of this it follows that the existence of non-terminating non-repeating decimals would guarantee that there are numbers which are not rationals. One such number was hinted upon earlier, namely, .1010010001.... You can write many more numbers like this. The numbers

.2020020002....

.120120012000120000....

.330004400005500000....

are all non-terminating non-repeating decimals.

In view of the above, a real number is sometimes defined as an infinite decimal. Remember that a terminating decimal, such as 3.45 can be looked upon as an infinite decimal 3.449. Therefore, every rational is an infinite decimal.

Notice that not all irrational numbers are of the form $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ... There are many irrational numbers which are very important but of a completely different nature. The numbers e and π are two such irrational numbers. When expressed as infinite decimals, their expressions are

$$e = 2.7182818284...$$

$$\text{and } \pi = 3.1415926535...$$

You will learn more about these numbers later.

Thus, starting from the counting numbers 1, 2, 3, ... that is the system N of natural numbers, we extended our system of numbers to the system Z of integers. To make up for the deficiencies of Z , we talked of the system Q of rational numbers. Even the system Q was found to have gaps.

In an attempt to fill the gaps, the system $\mathbf{R}$ of real numbers was created (See 1.7). $\mathbf{R}$ is one of the most beautiful mathematical systems. In this system, there are no gaps on the number line. To every point on the line there corresponds a real number and to every real number, there corresponds a point on the line. Thus, the collection of all points on the number line can be thought of as the system of real numbers.

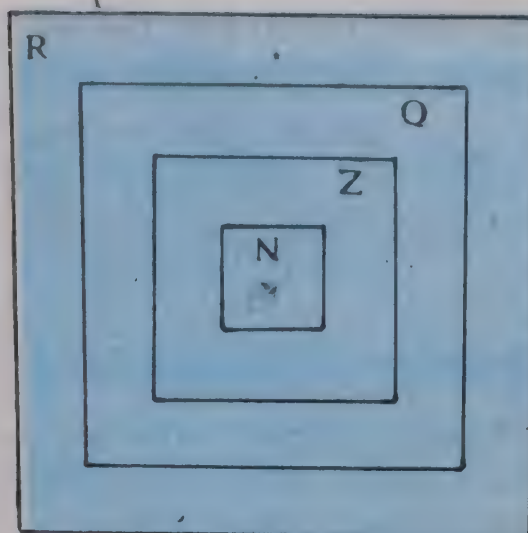


Fig. 1.7

Although $\mathbf{R}$ is a fairly satisfactory system of numbers having several pleasant properties, there is scope for further extension. We remarked earlier that the square root of a negative number is not possible. By adding new types of numbers to $\mathbf{R}$, we can get a number system called the system of complex numbers in which every real number has a square root. You will read about these numbers in higher classes.

EXERCISE 1(j)

1. Obtain decimal representation for

$$\frac{1}{2}, \frac{1}{3}, \frac{4}{9}, \frac{8}{13}, \frac{225}{16}$$

2. Write $56.23\overline{9}$ as a rational number.
3. Give an example of a non-terminating non-repeating decimal.
4. Give a construction to show that there is a point on the number line which represents the number $\sqrt{8}$.
5. Give an example to show that the sum of two irrational numbers is not always irrational.
6. Give an example to show that the product of two irrational numbers is not always irrational.

7. Look at Fig. 1.8 and continue the construction to show lengths representing

$$\sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, \\ \sqrt{9}, \sqrt{10}$$

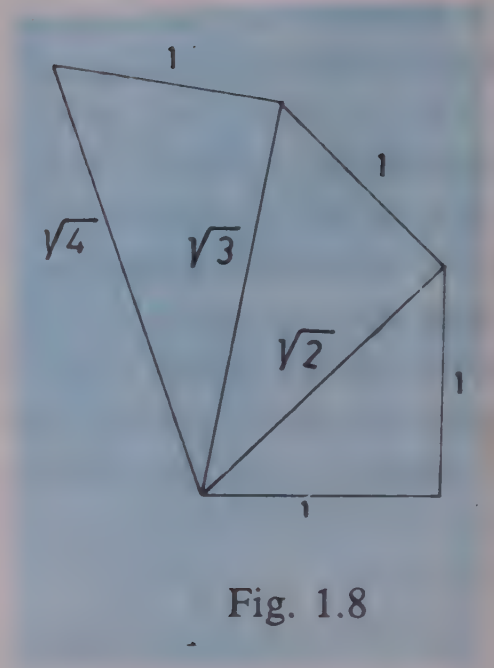


Fig. 1.8

Things to Remember

1. A natural number n is a perfect square if $n = m^2$, for some natural number m .
2. A number ending in 2, 3, 7 or 8 is never a perfect square.
3. The number of zeros in the end of a perfect square is never odd.
4. Squares of even numbers are always even.
5. Squares of odd numbers are always odd.
6. The square of a number (other than 1) is either a multiple of 3 or exceeds a multiple of 3 by 1.
7. The square of a number (other than 1) is either a multiple of 4 or exceeds a multiple of 4 by 1.
8. There are no natural numbers p and q such that $p^2 = 2q^2$.
9. The square root of a given natural number n is that natural number which when multiplied by itself gives n as the product.
10. The square root of a rational number m is that rational number which when multiplied by itself gives the given number.
11. If a and b are perfect squares, then

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

$$\text{and } \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

12. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ...etc. are irrationals.
13. The numbers e and π are irrational numbers which are not of the form $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ...

CHAPTER TWO

Cubes and Cube Roots

2.1 Cubes

LET US RECALL that

$$2^3 = 2 \times 2 \times 2 = 8$$

$$3^3 = 3 \times 3 \times 3 = 27$$

$$4^3 = 4 \times 4 \times 4 = 64$$

The above statements can also be expressed by saying that the *cube* of 2 is 8, the cube of 3 is 27, and the cube of 4 is 64. That is, the *cube* of a number is that number 'raised to the power 3'.

The following table gives the cubes of the first ten natural numbers :

Table 2.1

Number a	1	2	3	4	5	6	7	8	9	10
Cube a^3	1	8	27	64	125	216	343	512	729	1000

The numbers 1, 8, 27, ..., 1000 in the above table are cubes of the numbers 1, 2, 3, ..., 10 respectively. Such numbers as 1, 8, 27, ..., 1000 are called *perfect cubes*. Thus,

A natural number is said to be a *perfect cube* if it is the cube of some natural number. That is, a natural number n is a perfect cube if $n = m \times m \times m = m^3$, where m is a natural number.

Example 1 : Is 128 a perfect cube ?

Solution :

$$128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$$

From the prime factors of 128, we find that if we try to group together triples of equal factors, we are left with a single factor 2. Therefore, 128 is not a perfect cube.

2	128
2	64
2	32
2	16
2	8
2	4
	2

Example 2 : Is 27000 a perfect cube ?

Solution : If we resolve 27000 into prime factors, we have

$$27000 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$$

We find that the prime factors of 27000 can be grouped into triples of equal factors (as shown above) and no factor is left over.

Therefore, 27000 is a perfect cube. It is the cube of 30.
 $30 \times 30 \times 30 = 27000$

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
	5

Example 3 : (a) What is the smallest number by which 675 may be multiplied so that the product is a perfect cube ?

What is the smallest number by which 675 may be divided so that the quotient is a perfect cube ?

Solution : (a) By writing 675 as a product of its prime factors, we have

$$675 = \underline{3 \times 3 \times 3} \times 5 \times 5$$

We note that 3 occurs as a prime factor of 675 three times but 5 occurs as a prime factor only twice.

If we multiply 675 by 5, the product would be $\underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$, which is a perfect cube.

3	675
3	225
3	75
5	25
	5

Therefore, we must multiply 675 by 5 so that the product becomes a perfect cube.

(b) If we divide 675 by $5 \times 5 (=25)$, the quotient would be $3 \times 3 \times 3$ which is a perfect cube. Therefore, we must divide 675 by 25 so that the quotient is a perfect cube.

EXERCISE 2(a)

1. Find the cubes of the following numbers :
7, 12, 21, 100, 302
2. Write cubes of all natural numbers between 1 and 20 and verify the following statements :
 - (a) Cubes of all odd natural numbers are odd.
 - (b) Cubes of all even natural numbers are even.
3. Write cubes of 5 natural numbers which are multiples of 3 and verify the following:
'The cube of a natural number which is a multiple of 3 is a multiple of 27'.
4. Write cubes of 5 natural numbers which are of the form $3n + 1$ (e.g. 4, 7, 10, ...) and verify the following:
'The cube of a natural number of the form $3n + 1$ is a natural number of the same form'.
5. Write cubes of 5 natural numbers which are of the form $3n + 2$ (e.g. 5, 8, 11, ...) and verify the following:
'The cube of a natural number of the form $3n + 2$ is a natural number of the same form'.
6. Which of the following numbers are perfect cubes ?
64, 216, 243, 1728, 106480
7. What is the smallest number by which 392 must be multiplied so that the product is a perfect cube ?
8. What is the smallest number by which 8640 must be divided so that the quotient is a perfect cube ?
9. If one side of a cube is 13 metres, find its volume.
- *10. Use your calculator to write cubes of all natural numbers between 20 and 40.

Cube Roots

We have seen that the numbers 8, 343, 1331 are perfect cubes. That is, $8 = 2^3$, $343 = 7^3$, $1331 = 11^3$.

We say that 2 is the *cube root* of 8. Similarly, 7 is the cube root of 343, and 11 is the cube root of 1331.

Thus,

A natural number m is the cube root of a number n if $n = m^3$.

It follows that if m is the cube root of n , then n is the cube of m . Thus, in Example 2.2, 27 is the cube of 3 and, therefore, 3 is the cube root of 27.

Similarly, 729 is the cube of 9 and, therefore, 9 is the cube root of 729. The cube root of a number is indicated by the symbol ' $\sqrt[3]{}$ '. For example, the cube root of 8 is written as $\sqrt[3]{8}$. Thus, $\sqrt[3]{8} = 2$.

Similarly, $\sqrt[3]{343} = 7$, $\sqrt[3]{1331} = 11$.

Thus, for any perfect cube x , the cube root of x is denoted by the symbol $\sqrt[3]{x}$.

Here, for any number x , ' $\sqrt[3]{x}$ ' is called a *radical*, x is called the *radicand*, and 3 is called the *index* of the radical.

Remark : Observe that the symbol ' $\sqrt[3]{}$ ' for cube root is very much similar to the symbol for square root. The only difference is that whereas in the case of square root we use the symbol ' $\sqrt{}$ ', in the case of cube root we use the same ' $\sqrt{}$ ' but with a '3' as a superscript which indicates that we are taking a cube root. Strictly speaking, we should use the symbol ' $\sqrt[3]{}$ ' for cube root and ' $\sqrt[2]{}$ ' for square root. The '2' in this case is omitted for the sake of convenience.

Table 2.2

a	a^3	a	$\sqrt[3]{a}$
1	1	1	1
2	8	8	2
3	27	27	3
4	64	64	4
5	125	125	5
6	216	216	6
7	343	343	7
8	512	512	8
9	729	729	9
10	1000	1000	10

2.3 Cube Root Through a Pattern

We have :

$$\begin{aligned} 1^3 &= 1 \\ 2^3 - 1^3 &= 7 \\ 3^3 - 2^3 &= 19 \\ 4^3 - 3^3 &= 37 \\ 5^3 - 4^3 &= 61 \\ 6^3 - 5^3 &= 91, \text{ etc.} \end{aligned}$$

Also,

$$\begin{aligned} 1 &= 1^3 \\ 1 + 7 &= 2^3 \\ 1 + 7 + 19 &= 3^3 \\ 1 + 7 + 19 + 37 &= 4^3 \\ 1 + 7 + 19 + 37 + 61 &= 5^3, \text{ etc.} \end{aligned}$$

Thus, to find the cube root of a number, we have to subtract consecutive 1, 7, 19, 37, ... For example,

$$\begin{array}{r} 216 \\ \underline{1} \\ 215 \\ \underline{7} \\ 208 \\ \underline{19} \\ 189 \\ \underline{37} \\ 152 \\ \underline{61} \\ 91 \\ \underline{91} \\ 0 \end{array}$$

Since we had to subtract six times, therefore, $\sqrt[3]{216} = 6$.

The above rule can be used to calculate cube roots of small numbers.

To Find the Cube Root of a Perfect Cube by Factors

Look at the following carefully :

$$216 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}, \therefore \sqrt[3]{216} = 2 \times 3 = 6.$$

$$1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}, \therefore \sqrt[3]{1728} = 2 \times 2 \times 3 = 12.$$

$$1157625 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}, \therefore \sqrt[3]{1157625} = 3 \times 5 \times 7 = 105.$$

It is clear from the above examples that we have the following rule for finding the cube root of a perfect cube :

- (1) Write down the prime factors of the given number.
- (2) You will find that you are able to group these factors in an exact number of triples such that all the three factors in each triple are equal. Write one factor from each triple so formed.
- (3) Find the product of such factors.
- (4) The product is the required cube root.

Example 1 : Find the cube root of 157464.

Solution : We have :

$$157464 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$$

Therefore,

$$\sqrt[3]{157464} = 2 \times 3 \times 3 \times 3 = 54.$$

2	157464
2	78732
2	39366
3	19683
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
	3

Example 2 : Find the smallest number which when multiplied with 3600 will make the product a perfect cube. Further, find the cube root of the product.

Solution : When we resolve 3600 into its prime factors, we have

$$3600 = \underline{2 \times 2 \times 2} \times 2 \times 3 \times 3 \times 5 \times 5$$

Now, we know that if a number is to be a perfect cube then each of its prime factors must occur thrice or in multiples of three. Hence, the smallest number by which the given number must be multiplied in order that the product is a perfect cube is $2 \times 2 \times 3 \times 5 = 60$. Also, the product $= 3600 \times 60 = 216000$.

Arranging into triples of equal prime factors, the product

$$216000 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$$

$$\begin{aligned} \text{Therefore, } \sqrt[3]{216000} &= 2 \times 2 \times 3 \times 5 \\ &= 60 \end{aligned}$$

2	36
2	18
2	9
2	4
3	2
3	
5	

EXERCISE 2(b)

- Find the cube root of :
 (i) 343 (ii) 1000 (iii) 2744
 (iv) 74088 (v) 1000000
- Find the cube root of :
 (i) 134217728 (ii) 48228544 (iii) 74088 000
- Find the cube root of 125 through the pattern of Section 2.3.
- Multiply 137592 by the smallest number so that the product is a perfect cube. Also, find the cube root of the product.
- Divide the number 26244 by the smallest number so that the quotient is a perfect cube. Also, find the cube root of the quotient.
- The volume of a cube is 512 cubic metres. Find the length of the side of the cube.
- *7. Use your calculator to solve problem 2.

5 Cubes of Negative Integers

Since $(-1)^3 = -1$, therefore, the negative integer -1 is the cube of -1 . Similarly, $(-2)^3 = -8$ and, therefore, -8 is the cube of the negative integer -2 .

Table 2.3 shows the cubes of all negative integers $-1, -2, \dots, -10$. Thus, $-1, -8, -27, -64, -125, -216, -343, -512, -729, -1000$ are all cubes of negative integers. However, -100000 is not a cube of a number as $-100000 = (-10) \times (-10) \times (-10) \times (-10) \times (-10)$.

If m is any positive integer, then $-m^3$ is the cube of a negative integer. It is the cube of the negative integer $-m$. In fact, $(-m)^3 = (-m) \times (-m) \times (-m) = -m^3$.

6 Cube Root of a Negative Integer

We have noticed that if $-x$ ($x > 0$) is any negative integer, then

$$(-x)^3 = -x^3.$$

The above result can be expressed by saying that the cube root of $-x^3$ is $-x$.

That is, $\sqrt[3]{-x^3} = -x$.

Example 1 : Find the cube root of -1331 .

Solution : We have :

$$\begin{aligned} -1331 &= (-1) \times 11 \times 11 \times 11 \\ &= (-1)^3 \times 11^3 \\ &= (-1 \times 11)^3 = (-11)^3 \end{aligned}$$

$$\text{Therefore, } \sqrt[3]{-1331} = -11$$

Remark : Observe that $|-1331|$ is the positive integer 1331 whose cube root is 11 . To obtain the cube root of -1331 , we find the cube root of its absolute value and multiply it by -1 . The product is the desired cube root.

The above example suggests the following rule :

If m be a positive integer, then $\sqrt[3]{-m^3} = -\sqrt[3]{m^3}$.

Example 2 : Find the cube root of -1728 .

$$\text{Solution : } \sqrt[3]{-1728} = -\sqrt[3]{1728}$$

Table 2.3

x	x^3
-1	-1
-2	-8
-3	-27
-4	-64
-5	-125
-6	-216
-7	-343
-8	-512
-9	-729
-10	-1000

Let us, therefore, first find the cube root of 1728.

$$1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$$

$$\therefore \sqrt[3]{1728} = 2 \times 2 \times 3 \\ = 12$$

$$\text{Hence, } \sqrt[3]{-1728} = -\sqrt[3]{1728} \\ = -12$$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

EXERCISE 2 (c)

1. Which of the following numbers are cubes of negative integers ?

- (a) -64 (b) -2197 (c) -1056 (d) -3888

2. Find the cube roots of :

- (a) -125 (b) -5832 (c) -17576 (d) -2744000

2.7 Cube Root of a Product

We know that if a and b are two positive integers, then

$$(ab)^2 = a^2b^2$$

and $\sqrt{ab} = \sqrt{a} \sqrt{b}$

Similarly, $(ab)^3 = a^3b^3$

It is natural to ask whether a similar relation holds for cube roots as well.

That is, can we say that the relation $\sqrt[3]{ab} = \sqrt[3]{a} \sqrt[3]{b}$ is true ?

Let us consider the following examples :

Example 1 : Find the value of each of the following :

(a) $\sqrt[3]{27 \times 64}$

$$(b) \sqrt[3]{27} \times \sqrt[3]{64}$$

Do you observe anything ?

Solution : (a) Since $27 \times 64 = \underline{3 \times 3 \times 3} \times \underline{4 \times 4 \times 4}$,

Therefore, $\sqrt[3]{27 \times 64} = 3 \times 4 = 12$

$$b) \sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$$

$$\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$$

$$\therefore \sqrt[3]{27} \times \sqrt[3]{64} = 3 \times 4 = 12$$

The value is the same in both the cases.

Therefore,

$$\sqrt[3]{27 \times 64} = \sqrt[3]{27} \times \sqrt[3]{64}$$

Example 2 : Show that

$$(a) \sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729}$$

$$(b) \sqrt[3]{(-125) \times 216} = \sqrt[3]{-125} \times \sqrt[3]{216}$$

$$(c) \sqrt[3]{343 \times (-512)} = \sqrt[3]{343} \times \sqrt[3]{-512}$$

$$(d) \sqrt[3]{(-125) \times (-1000)} = \sqrt[3]{-125} \times \sqrt[3]{-1000}$$

Solution : (a) Since $64 \times 729 = \underline{4 \times 4 \times 4} \times \underline{9 \times 9 \times 9}$,

therefore, $\sqrt[3]{64 \times 729} = 4 \times 9$

$$= \sqrt[3]{64} \times \sqrt[3]{729}$$

(b) Since $(-125) \times 216 = -(125 \times 216)$

$$= -(\underline{5 \times 5 \times 5} \times \underline{6 \times 6 \times 6}),$$

therefore, $\sqrt[3]{(-125) \times 216} = -(5 \times 6)$

$$= (-5) \times 6$$

$$= \sqrt[3]{-125} \times \sqrt[3]{216}$$

$$\begin{aligned} (c) \text{ Since } 343 \times (-512) &= - (343 \times 512) \\ &= - (7 \times 7 \times 7 \times 8 \times 8 \times 8), \end{aligned}$$

$$\begin{aligned} \text{therefore, } \sqrt[3]{343 \times (-512)} &= - (7 \times 8) \\ &= 7 \times (-8) \\ &= \sqrt[3]{343} \times \sqrt[3]{-512} \end{aligned}$$

$$\begin{aligned} (d) \text{ Since } (-125) \times (-1000) &= 125 \times 1000 \\ &= 5 \times 5 \times 5 \times 10 \times 10 \times 10 \end{aligned}$$

$$\begin{aligned} \text{therefore, } \sqrt[3]{(-125) \times (-1000)} &= 5 \times 10 \\ &= (-5) \times (-10) \\ &= \sqrt[3]{-125} \times \sqrt[3]{-1000} \end{aligned}$$

The above examples suggest that the cube root of the product of two integers is the product of their cube roots. We, therefore, have the following rule :

If a and b are any two integers, then

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \quad (\text{I})$$

Remark : Observe that whereas the above rule holds for all integers a and b , the corresponding rule for square roots holds only when a and b are both positive integers. (Recall that the square root of a negative number does not exist.)

EXERCISE 2(d)

Find the cube root of each of the following numbers using rule (I) :

1. 8×64

2. $(-216) \times 1728$

3. $27 \times (-2744)$

4. $(-125) \times (-3375)$

5. $(729) \times 15625$

6. -456533

7. -474552

8. -5832000

*9. Use your calculator to solve problems 1 to 8.

2.8 Cubes of Rational Numbers

Just as we found the cubes of natural numbers and negative integers, similarly, we can find the cubes of rational numbers which are not integers.

For example, $\left(\frac{4}{7}\right)^3 = \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} = \frac{64}{343}$

$$(.06)^3 = .06 \times .06 \times .06 = .000216$$

In other words, the cube of $\frac{4}{7}$ is $\frac{64}{343}$, and the cube of .06 is .000216.

Let us recall that if $\frac{p}{q}$ is a rational number, then $\left(\frac{p}{q}\right)^3 = \frac{p^3}{q^3}$.

This gives us a simple method for finding the cube of a rational number.

Example 1 : Find the cube of $\frac{5}{9}$.

Solution : $\left(\frac{5}{9}\right)^3 = \frac{5^3}{9^3} = \frac{5 \times 5 \times 5}{9 \times 9 \times 9} = \frac{125}{729}$

Example 2 : Write $\frac{27}{125}$ in power notation. Is it the cube of a rational number ?

Solution : $\frac{27}{125} = \frac{3 \times 3 \times 3}{5 \times 5 \times 5} = \frac{3^3}{5^3} = \left(\frac{3}{5}\right)^3$

It is clear, from the above, that $\frac{27}{125}$ is the cube of the rational number $\frac{3}{5}$.

Example 3 : Find the cube of 3.1.

Solution : $(3.1)^3 = 3.1 \times 3.1 \times 3.1$
 $= 29.791$

2.9 Cube Root of a Rational Number

If x is a rational number such that $x^3 = a$, then we say that x is the cube root of a .

For example,

$$\left(\frac{3}{4}\right)^3 = \frac{27}{64}, \text{ therefore, the cube root of } \frac{27}{64} \text{ is } \frac{3}{4}.$$

Similarly, $\left(\frac{-5}{11}\right)^3 = \frac{-125}{1331}$, therefore, the cube root of $\frac{-125}{1331}$ is $\frac{-5}{11}$.

2.10 To Find the Cube Root of a Rational Number

We know that if a and b be two positive integers, then

$$\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

How about a similar equality for cube roots? We should expect that the following equality is also true :

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

Consider the following example :

Example 4 : Find the value of each of the following :

$$(a) \sqrt[3]{\frac{27}{64}}$$

$$(b) \frac{\sqrt[3]{27}}{\sqrt[3]{64}}$$

Do you observe anything ?

$$\begin{aligned} \text{Solution : } (a) \quad \frac{27}{64} &= \frac{3 \times 3 \times 3}{4 \times 4 \times 4} \\ &= \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \\ &= \left(\frac{3}{4}\right)^3 \end{aligned}$$

$$\text{Therefore, } \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$$

$$(b) \sqrt[3]{27} = \sqrt[3]{3 \times 3 \times 3} = 3$$

$$\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$$

$$\therefore \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$$

The values are the same in both the cases.

$$\text{Therefore, } \sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}}$$

Example 5 : Show that

$$(a) \sqrt[3]{\frac{64}{125}} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}}$$

$$(b) \sqrt[3]{\frac{-27}{512}} = \frac{\sqrt[3]{-27}}{\sqrt[3]{512}}$$

$$(c) \sqrt[3]{\frac{125}{-343}} = \frac{\sqrt[3]{125}}{\sqrt[3]{-343}}$$

$$(d) \sqrt[3]{\frac{-343}{-729}} = \frac{\sqrt[3]{-343}}{\sqrt[3]{-729}}$$

Solution :

$$a) \frac{64}{125} = \frac{4 \times 4 \times 4}{5 \times 5 \times 5} = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \left(\frac{4}{5}\right)^3$$

$$\text{Therefore, } \sqrt[3]{\frac{64}{125}} = \frac{4}{5} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}}$$

$$b) \frac{-27}{512} = \frac{(-3) \times (-3) \times (-3)}{8 \times 8 \times 8}$$

$$= \left(\frac{-3}{8}\right) \times \left(\frac{-3}{8}\right) \times \left(\frac{-3}{8}\right) = \left(\frac{-3}{8}\right)^3$$

$$\text{Therefore, } \sqrt[3]{\frac{-27}{512}} = \frac{-3}{8} = \frac{\sqrt[3]{-27}}{\sqrt[3]{512}}$$

$$(c) \quad \frac{125}{-343} = \frac{5 \times 5 \times 5}{(-7) \times (-7) \times (-7)} = \frac{5}{-7} \times \frac{5}{-7} \times \frac{5}{-7} = \left(\frac{5}{-7}\right)^3$$

$$\text{Therefore, } \sqrt[3]{\frac{125}{-343}} = \frac{5}{-7} = \frac{\sqrt[3]{125}}{\sqrt[3]{-343}}$$

$$(d) \quad \frac{-343}{-729} = \frac{343}{729} = \frac{7 \times 7 \times 7}{9 \times 9 \times 9} = \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} = \left(\frac{7}{9}\right)^3$$

$$\text{Therefore, } \sqrt[3]{\frac{-343}{-729}} = \frac{7}{9} = \frac{-7}{-9}$$

$$= \frac{\sqrt[3]{(-7) \times (-7) \times (-7)}}{\sqrt[3]{(-9) \times (-9) \times (-9)}}$$

$$= \frac{\sqrt[3]{-343}}{\sqrt[3]{-729}}$$

The above examples suggest the following :

If a and b are any integers, b ≠ 0, then

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}} \quad (\text{II})$$

EXERCISE 2(e)

121211
-121211

121211
-121211

121211
-121211

121211

1. Find the cube of :

(a) $\frac{7}{9} \frac{343}{729}$

(b) $\frac{-8}{11} = \frac{512}{1331}$

(c) $\frac{12}{7} = \frac{1728}{343}$

(d) $\frac{-13}{8} = \frac{-2197}{512}$

(e) $2\frac{3}{5} = \frac{2197}{125}$

(f) $3\frac{1}{4} = \frac{8197}{64}$

2. Find the cube of :

(a) .3

(b) 1.5

(c) .08

(d) 2.1

3. Find which of the following numbers are cubes of rational numbers:

(a) $\frac{27}{64} = \frac{3}{4}$

(b) $\frac{125}{128} = \frac{5}{8}$

(c) .001331

(d) .04

4. Use rule (II) to find the cube root of each of the following numbers:

(a) $\frac{8}{125} = \frac{2}{5}$

(b) $\frac{-64}{1331} = \frac{-4}{11}$

(c) $\frac{27}{-4096} = \frac{3}{-16}$

(d) $\frac{-2197}{-9261} = \frac{13}{21}$

5. The volume of a cubical box is 32.768 cubic metres. Find the length of a side of the box.

The Use of Cube Root Tables to Determine Approximate Values

We have seen that not all numbers are perfect cubes. For example, if you look at Table 2.1, you will see that from 1 to 1000, there are only ten natural numbers which are perfect cubes, namely, 1, 8, 27, ..., 512, 729 and 1000. The remaining 990 natural numbers are *not* perfect cubes. There is no rational number whose cube is 2. There is no rational number whose cube is 3 or 4 or 5. $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{4}$ are all irrational numbers. If a number is not a perfect cube, then we try to find a rational number whose cube is nearly equal to the given number. This enables us to find an approximate value of the cube root of the number. There are many practical situations in which an approximate value of the cube root of a number is useful. For example, we may be interested in finding the length of a side of a cube whose volume is 7 cubic metres. In other words, we want to find the value of $\sqrt[3]{7}$.

Since $1^3 < 7 < 2^3$, we know that

$$1 < \sqrt[3]{7} < 2$$

Now, 1.1, 1.2, 1.3, ..., 1.9 are all numbers between 1 and 2. If we calculate cubes of these numbers, we find that

$$(1.9)^3 < 7 < 2^3$$

$$\therefore 1.9 < \sqrt[3]{7} < 2$$

Thus, 1.9 is nearer to $\sqrt[3]{7}$ than 1 is.

Again, if we calculate cubes of numbers 1.91, 1.92, ..., 1.99, we find that

$$(1.91)^3 < 7 < (1.92)^3$$

$$\therefore 1.91 < \sqrt[3]{7} < 1.92$$

It follows that 1.91 is still nearer to $\sqrt[3]{7}$ than 1.9 is.

We can continue this process indefinitely to get a better and better approximation to $\sqrt[3]{7}$. Cube root of 7 being an infinite decimal, we can calculate its value to a desired number of decimal places. (If you have a calculator, you may use it to find values of cubes of numbers required in the process.) However, this method of finding approximate values of cube roots is very cumbersome. For this reason, tables have been prepared giving approximate values of cube roots of different numbers. Cube roots of most numbers can be written down just by looking at these tables. Let us learn to use these tables. Look at Table 2.4. It gives values of cube roots of $\sqrt[3]{10x}$, $\sqrt[3]{100x}$, for all natural numbers x from 1 to 99.

Example 1 : Find the value of $\sqrt[3]{7}$, $\sqrt[3]{70}$ and $\sqrt[3]{700}$ from the table.

Solution : To find the required values, look at the row containing 7 in the column of x . Read values horizontally in this row in the columns $\sqrt[3]{x}$, $\sqrt[3]{10x}$, $\sqrt[3]{100x}$ respectively.

Thus,

For $x = 7$,

$$\sqrt[3]{7} = 1.913, \sqrt[3]{70} = \sqrt[3]{10 \times 7} = 4.121,$$

$$\sqrt[3]{700} = \sqrt[3]{100 \times 7} = 8.879$$

Table 2.4

$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$	x	$\sqrt[3]{x}$	$\sqrt[3]{10x}$	$\sqrt[3]{100x}$
1.000	2.154	4.642	50	3.684	7.937	17.10
1.260	2.714	5.848	51	3.708	7.990	17.21
1.442	3.107	6.694	52	3.733	8.041	17.32
1.587	3.420	7.368	53	3.756	8.093	17.44
1.710	3.684	7.937	54	3.780	8.143	17.54
1.817	3.915	8.434	55	3.803	8.193	17.65
1.913	4.121	8.879	56	3.826	8.243	17.76
2.000	4.309	9.283	57	3.849	8.291	17.86
2.080	4.481	9.655	58	3.871	8.340	17.97
2.154	4.642	10.00	59	3.893	8.387	18.07
2.224	4.791	10.32	60	3.915	8.434	18.17
2.289	4.932	10.63	61	3.936	8.481	18.27
2.351	5.066	10.91	62	3.958	8.527	18.37
2.410	5.192	11.19	63	3.979	8.573	18.47
2.466	5.313	11.45	64	4.000	8.618	18.57
2.520	5.429	11.70	65	4.021	8.662	18.66
2.571	5.540	11.93	66	4.041	8.707	18.76
2.621	5.646	12.16	67	4.062	8.750	18.85
2.668	5.749	12.39	68	4.082	8.794	18.95
2.714	5.848	12.60	69	4.102	8.837	19.04
2.759	5.944	12.81	70	4.121	8.879	19.13
2.802	6.037	13.01	71	4.141	8.921	19.22
2.844	6.127	13.20	72	4.160	8.963	19.31
2.884	6.214	13.39	73	4.179	9.004	19.40
2.924	6.300	13.57	74	4.198	9.045	19.49
2.962	6.383	13.75	75	4.217	9.086	19.57
3.000	6.463	13.92	76	4.236	9.126	19.66
3.037	6.542	14.09	77	4.254	9.166	19.75
3.072	6.619	14.26	78	4.273	9.205	19.83
3.107	6.694	14.42	79	4.291	9.244	19.92
3.141	6.768	14.58	80	4.309	9.283	20.00
3.175	6.840	14.74	81	4.327	9.322	20.08
3.208	6.910	14.89	82	4.344	9.360	20.17
3.240	6.980	15.04	83	4.362	9.398	20.25
3.271	7.047	15.18	84	4.380	9.435	20.33
3.302	7.114	15.33	85	4.397	9.473	20.41
3.332	7.179	15.47	86	4.414	9.510	20.49
3.362	7.243	15.60	87	4.431	9.546	20.57
3.391	7.306	15.74	88	4.448	9.583	20.65
3.420	7.368	15.87	89	4.465	9.619	20.72
3.448	7.429	16.01	90	4.481	9.655	20.80
3.476	7.489	16.13	91	4.498	9.691	20.88
3.503	7.548	16.26	92	4.514	9.726	20.95
3.530	7.606	16.39	93	4.531	9.761	21.03
3.557	7.663	16.51	94	4.547	9.796	21.10
3.583	7.719	16.63	95	4.563	9.830	21.18
3.609	7.775	16.75	96	4.579	9.865	21.25
3.634	7.830	16.87	97	4.595	9.899	21.33
3.659	7.884	16.98	98	4.610	9.933	21.40
			99	4.626	9.967	21.47

Example 2 : Find the cube root of

(a) 73.8

(b) 732

(c) 7342

Solution :

(a) Since $73 < 73.8 < 74$,

$$\sqrt[3]{73} < \sqrt[3]{73.8} < \sqrt[3]{74}$$

From the cube root table,

$$\sqrt[3]{74} = 4.198$$

$$\sqrt[3]{73} = 4.179$$

Difference for $(74 - 73)$ i.e. 1 = $(4.198 - 4.179) = .019$

$\therefore$ Difference for .8 = $.019 \times .8 = .0152 = .015$ (to 3 places of decimal)

$$\sqrt[3]{73} = 4.179$$

$$\sqrt[3]{73.8} = \underline{\underline{4.194}}$$

(b) $730 < 732 < 740$

$$\therefore \sqrt[3]{730} < \sqrt[3]{732} < \sqrt[3]{740}$$

As in (a), since $740 - 730 = 10$,

$$\sqrt[3]{732} = \sqrt[3]{730} + \frac{2}{10} \times (\sqrt[3]{740} - \sqrt[3]{730})$$

$$= 9.004 + \frac{1}{5} \times (9.045 - 9.004)$$

$$= 9.004 + \frac{1}{5} \times (.041)$$

$$= 9.004 + .008 = 9.012$$

(c) $\sqrt[3]{7300} < \sqrt[3]{7342} < \sqrt[3]{7400}$

$$\therefore \sqrt[3]{7342} = \sqrt[3]{7300} + \frac{42}{100} \times (\sqrt[3]{7400} - \sqrt[3]{7300})$$

$$\begin{aligned}
 &= 19.40 + .42 \times (19.49 - 19.40) \\
 &= 19.40 + .42 \times .09 \\
 &= 19.40 + .038 \\
 &= 19.438
 \end{aligned}$$

Remark : As in the case of square roots, all the above values, strictly speaking, are approximate only. We should, therefore, use ' $\approx$ ' instead of '=' everywhere. However, there is no harm in using '=', if we remember that the values are only approximate.

Example 3 : Find the value of $\sqrt[3]{792}$.

$$\begin{aligned}
 \text{Solution : } \sqrt[3]{792} &= \sqrt[3]{8 \times 99} \\
 &= \sqrt[3]{8} \times \sqrt[3]{99} \\
 &= 2 \times 4.626 \\
 &= 9.252
 \end{aligned}$$

Example 4 : Find the cube root of 29600.

$$\begin{aligned}
 \text{Solution : } \sqrt[3]{29600} &= \sqrt[3]{100 \times 8 \times 37} \\
 &= \sqrt[3]{8} \times \sqrt[3]{100 \times 37} \\
 &= 2 \times 15.47 \\
 &= 30.94
 \end{aligned}$$

Example 5 : Find the cube root of $\frac{31}{216}$.

$$\begin{aligned}
 \text{Solution : } \sqrt[3]{\frac{31}{216}} &= \frac{\sqrt[3]{31}}{\sqrt[3]{216}} \\
 &= \frac{1}{6} \times \sqrt[3]{31} \\
 &= \frac{1}{6} \times 3.141 \\
 &= .5235
 \end{aligned}$$

Remark : From the above examples, it is clear that using the rules

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b} \text{ and } \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}},$$

we can write approximate values of cube roots of several other numbers whose values are not listed in the table.

EXERCISE 2 (f)

Making use of the cube root table, find the cube roots of the following (correct to three decimal places) :

1. $\sqrt[3]{13}$

2. $\sqrt[3]{250}$

3. $\sqrt[3]{1100}$

4. $\frac{29}{729}$

5. $\frac{51}{125000}$

6. $\sqrt[3]{5112}$

7. $\sqrt[3]{17064}$

8. $\sqrt[3]{9800}$

9. $\sqrt[3]{309400}$

10. $\sqrt[3]{85.9}$

11. $\sqrt[3]{953}$

12. $\sqrt[3]{5319}$

13. Find the approximate length of the side of a cube whose volume is 275 cubic metres.

*14. Use your calculator to find the values of all cube roots in problems 1 to 12.

Things to Remember

1. A natural number n is a perfect cube if $n = m^3$, where m is a natural number.
2. A natural number m is the cube root of a number n , if $n = m^3$. For any perfect cube a , the cube root of a is written as $\sqrt[3]{a}$.

$\sqrt[3]{a}$ is called a radical, a is called the radicand, and 3 is called the index of the radical.

3. If a and b be any two integers, then

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

$$\text{and } \sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}, (b \neq 0)$$

4. $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{4}$ are all irrational numbers.

CHAPTER THREE

Exponents and Radicals

3.1 Rational Exponents

LET US RECALL that $2 \times 2 \times 2 \times 2 \times 2 \times 2$ can be written in the exponential or power notation as 2^6 . We read 2^6 as 'two raised to the power six' or as 'six power of two'. In 2^6 , the number '2' is called the base and '6' is called the exponent or the index. Similarly, in $(-5)^4$, the number '-5' is the base and 4 is the exponent. In $\left(\frac{3}{7}\right)^{-8}$, the number $\frac{3}{7}$ is the base and '-8' is the exponent. We have seen in class VII that it is often very convenient to use the exponential notation.

We have also learnt the following facts about exponents :

- (I) If x, y be any rational numbers different from zero and m be any integer then $x^m \times y^m = (x \times y)^m$.
- (II) If x be any rational number different from zero and m, n be any integers then $x^m \times x^n = x^{m+n}$
 $(x^m)^n = x^{m \times n}$

In all the facts stated above, the exponents are integers. A very natural question arises :

Can the exponent be a rational number other than an integer ? Can we give any meaning to expressions like

$$4^{\frac{1}{2}}, 9^{-\frac{1}{2}}, 16^{\frac{3}{4}}, 125^{-\frac{5}{3}} ?$$

We shall, herein after, find that every rational number can be an exponent

Reciprocals of Positive Integers as Exponents

We know that the statement $3^2 = 9$ can be written as $\sqrt{9} = 3$. We agree to write it in another way as $9^{\frac{1}{2}} = 3$ (and read it as 'nine raised to the power half is three').

As usual, here 9 is the base and $\frac{1}{2}$ is the exponent.

Similarly, $4^3 = 64$ can be written as $\sqrt[3]{64} = 4$, or $64^{\frac{1}{3}} = 4$.

We can go a little further and use the rational numbers $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, ... as exponents.

Thus,

$$3^4 = 81 \text{ can be written as } 81^{\frac{1}{4}} = 3.$$

Here, $81^{\frac{1}{4}}$ is called the fourth root of 81 and is written as $\sqrt[4]{81}$ as a radical.

Likewise, $4^5 = 1024$ can be written as $1024^{\frac{1}{5}} = 4$.

Here, $1024^{\frac{1}{5}}$ is called the fifth root of 1024 and is written as $\sqrt[5]{1024}$ as a radical.

Similarly, $\left(\frac{2}{3}\right)^6 = \frac{64}{729}$ can be written as $\sqrt[6]{\frac{64}{729}} = \frac{2}{3}$.

The above examples suggest that :

If q be any positive integer greater than 1, and x and y be rational numbers such that $x^q = y$, then $y^{\frac{1}{q}} = x$.

We write $y^{\frac{1}{q}}$ as $\sqrt[q]{y}$ and read it as q th root of y .

$\sqrt[q]{y}$ is called a *radical* and q is called the *index* of the radical.

EXERCISE 3(a)

1. Express the following in the exponential form :

$$\sqrt{5}, \sqrt[3]{7}, \sqrt[9]{1100}, \sqrt[4]{\frac{3}{4}}, \sqrt[8]{\frac{61}{1123}}$$

2. Write the following as radicals :

$$16^{\frac{1}{2}}, 125^{\frac{1}{3}}, \left(\frac{6}{17}\right)^{\frac{1}{9}}, \left(\frac{11}{17}\right)^{\frac{1}{11}}, \left(\frac{61}{325}\right)^{\frac{1}{17}}$$

3.3 Positive Rational Numbers as Exponents

Let $\frac{p}{q}$ be any positive rational number (where p and q are positive integers prime to each other) and let x be any rational number. We have already given a meaning to $x^{\frac{1}{q}}$.

We shall now give a meaning to $x^{\frac{p}{q}}$. This can be done very easily :

$$x^{\frac{p}{q}} \text{ is defined as } (x^p)^{\frac{1}{q}}.$$

That is, $x^{\frac{p}{q}}$ is the q th root of x^p .

Thus,

$$\begin{aligned} 4^{\frac{3}{2}} &= (4^3)^{\frac{1}{2}} = (64)^{\frac{1}{2}} = 8 \\ 8^{\frac{2}{3}} &= (8^2)^{\frac{1}{3}} = (64)^{\frac{1}{3}} = 4 \end{aligned}$$

3.4 Negative Rational Numbers as Exponents

We have given a meaning to $x^{\frac{p}{q}}$ where $\frac{p}{q}$ is a positive rational number. If $\frac{p}{q}$ a negative rational number, then $-\frac{p}{q}$ is a positive rational number and, in this case, $x^{\frac{p}{q}}$ ($x \neq 0$) can be written as $\frac{1}{x^{-\frac{p}{q}}}$, making the exponent a positive rational number.

$$\begin{aligned} \text{Thus, } 4^{-\frac{3}{2}} &= \frac{1}{4^{\frac{3}{2}}} = \frac{1}{8} \\ 8^{-\frac{2}{3}} &= \frac{1}{8^{\frac{2}{3}}} = \frac{1}{4} \text{ etc.} \end{aligned}$$

EXERCISE 3(b)

Find the value of :

1. $8^{\frac{1}{3}}$

2. $27^{\frac{2}{3}}$

3. $8^{\frac{5}{3}}$

4. $16^{-\frac{3}{4}}$

5. $216^{-\frac{2}{3}}$

6. $64^{-\frac{5}{2}}$

7. $125^{\frac{2}{3}}$

8. $(343)^{\frac{2}{3}}$

9. $(243)^{\frac{2}{5}}$

10. $(512)^{-\frac{2}{9}}$

Laws of Exponents for Rational Exponents

As stated on page 74 the laws of exponents for integral exponents.

These laws hold for rational exponents as well.

Consider the following examples :

$$9^{\frac{1}{2}} \times 4^{\frac{1}{2}} = 3 \times 2 = 6 = (36)^{\frac{1}{2}} = (9 \times 4)^{\frac{1}{2}}$$

$$5^{\frac{2}{3}} \times 5^{\frac{1}{3}} = (5^2)^{\frac{1}{3}} \times 5^{\frac{1}{3}} = (5^2 \times 5)^{\frac{1}{3}} = 5^1 = 5^{\frac{2}{3} + \frac{1}{3}}$$

$$\left((64)^{\frac{1}{3}}\right)^{\frac{1}{2}} = 4^{\frac{1}{2}} = 2 = (64)^{\frac{1}{6}} = (64)^{\frac{1}{3} \times \frac{1}{2}}$$

In view of the above examples, we have the following laws for rational exponents :

If x and y be any rational numbers different from zero and a be any rational number, then

$$x^a \times y^a = (x \times y)^a \quad \text{(III)}$$

If x be any rational number different from zero, and a, b be any rational numbers, then

$$\begin{aligned} x^a \times x^b &= x^{a+b} \\ (x^a)^b &= x^{a \times b} \end{aligned} \quad \text{(IV)}$$

Remark : We have defined $x^{\frac{p}{q}}$ as the number $\sqrt[q]{x^p}$. Notice, however, that if q is an even integer, then x^p must be positive. We have already remarked earlier that the square root of a negative number does not exist. For the same reason, q th root of a negative number does not exist if q is even.

Example 1 : Find the value of

$$81^{\frac{1}{3}} \times 576^{\frac{1}{3}}$$

Solution : We have

$$\begin{aligned} 81^{\frac{1}{3}} \times 576^{\frac{1}{3}} &= (81 \times 576)^{\frac{1}{3}} \\ &= [(3^2 \times 3^2) \times (4^3 \times 3^2)]^{\frac{1}{3}} \\ &= (3^2 \times 3^2 \times 3^2 \times 4^3)^{\frac{1}{3}} \\ &= (3^3 \times 3^3 \times 4^3)^{\frac{1}{3}} \\ &= \sqrt[3]{3^3 \times 3^3 \times 4^3} \\ &= 3 \times 3 \times 4 \\ &= 36 \end{aligned}$$

Example 2 : Find the value of

$$64^{-\frac{2}{3}} \times 27^{-\frac{2}{3}}$$

Solution : We have

$$\begin{aligned} 64^{-\frac{2}{3}} \times 27^{-\frac{2}{3}} &= (64 \times 27)^{-\frac{2}{3}} \\ &= (4^3 \times 3^3)^{-\frac{2}{3}} \\ &= (4 \times 3)^{-\frac{2}{3}} \\ &= (12^3)^{-\frac{2}{3}} = 12^{3 \times (-\frac{2}{3})} = 12^{-2} \\ &= \frac{1}{12^2} = \frac{1}{144} \end{aligned}$$

Example 3 : Evaluate $15^{\frac{1}{2}} \times 15^{\frac{1}{2}}$.

Solution : We have

$$15^{\frac{1}{2}} \times 15^{\frac{1}{2}} = 15^{\frac{1}{2} + \frac{1}{2}} = 15.$$

Remark : The above example illustrates why we agreed to write $\sqrt{x}$ as $x^{\frac{1}{2}}$. Similarly, $x^{\frac{1}{3}} \times x^{\frac{1}{3}} \times x^{\frac{1}{3}} = (x^{\frac{1}{3}} \times x^{\frac{1}{3}}) \times x^{\frac{1}{3}} = (x^{\frac{1}{3} + \frac{1}{3}}) \times x^{\frac{1}{3}} = x^{\frac{2}{3}} \times x^{\frac{1}{3}} = x^{\frac{2}{3} + \frac{1}{3}} = x^1 = x$. For this reason, we agreed to write $\sqrt[3]{x}$ as $x^{\frac{1}{3}}$. In general,

$$x^{\frac{1}{q}} \times x^{\frac{1}{q}} \times \dots \times x^{\frac{1}{q}} \quad (q \text{ times})$$

$$= x^{\frac{1}{q} + \frac{1}{q} + \dots + \frac{1}{q} (q \text{ times})} = x^{q \times \frac{1}{q}} = x^1 = x.$$

Therefore, we say that the q th root of x is $x^{\frac{1}{q}}$.

Example 4 : Evaluate $(64^{\frac{2}{3}})^{\frac{1}{2}}$.

Solution : We have

$$\begin{aligned} (64^{\frac{2}{3}})^{\frac{1}{2}} &= 64^{\frac{2}{3} \times \frac{1}{2}} = 64^{\frac{1}{3}} \\ &= (2 \times 2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{3}} \\ &= (2^6)^{\frac{1}{3}} \\ &= 2^{6 \times \frac{1}{3}} = 2^2 \\ &= 4 \end{aligned}$$

Example 5 : Evaluate $16^{\frac{5}{2}} \div 16^{\frac{1}{2}}$.

Solution : We have

$$\begin{aligned} 16^{\frac{5}{2}} \div 16^{\frac{1}{2}} &= \frac{16^{\frac{5}{2}}}{16^{\frac{1}{2}}} = 16^{\frac{5}{2}} \times 16^{-\frac{1}{2}} \\ &= 16^{\frac{5}{2} + (-\frac{1}{2})} \\ &= 16^{\frac{5}{2} - \frac{1}{2}} = 16^2 \\ &= 256 \end{aligned}$$

Remark : Example 5 above illustrates the following law :

$$\boxed{x^a \div x^b = x^{a-b}} \quad , \text{ since}$$

$$x^a \div x^b = \frac{x^a}{x^b} = x^a \times x^{-b} = x^{a + (-b)} = x^{a-b}$$

EXERCISE 3(c)

1. Express the following in the exponential form :

(a) $\sqrt[4]{37}$

(b) $\sqrt[5]{27}$

(c) $\sqrt[7]{29^2}$

(d) $\sqrt[6]{\frac{8}{9}}$

(e) $\sqrt[8]{\left(\frac{2}{3}\right)^2}$

(f) $\sqrt[3]{2^{-6}}$

2. Express as radicals :

(a) $5^{\frac{1}{4}}$

(b) $21^{\frac{2}{3}}$

(c) $2^{\frac{5}{6}}$

(d) $\left(\frac{5}{17}\right)^{\frac{1}{9}}$

(e) $\left(\frac{17}{21}\right)^{\frac{2}{5}}$

(f) $(-215)^{\frac{1}{7}}$

3. Find the value of :

(a) $32^{\frac{1}{5}}$

(b) $\frac{4}{36^{-\frac{1}{2}}}$

(c) $16^{\frac{-3}{4}}$

(d) $100^{\frac{-3}{2}}$

(e) $\left(\frac{1}{9}\right)^{\frac{-1}{2}}$

4. Simplify :

(a) $7^{\frac{1}{2}} \times 7^{\frac{3}{2}}$

(b) $11^{\frac{4}{3}} \times 11^{\frac{2}{3}}$

(c) $2 \times 9^{\frac{3}{2}} \times 9^{-\frac{1}{2}}$

(d) $27^{\frac{2}{3}} \times 27^{\frac{1}{3}} \times 27^{\frac{-4}{3}}$

5. Simplify :

(a) $13^{\frac{4}{3}} \div 13^{\frac{1}{3}}$

(b) $100^{\frac{3}{2}} \div 100^{\frac{1}{2}}$

(c) $\left(110^{\frac{1}{2}}\right)^4$

(d) $(5^{10})^0$

6. Evaluate :

(a) $3 \times 16^{\frac{3}{4}}$

(b) $2 \times (27)^{\frac{-2}{3}}$

(c) $5^2 \times 5^{-4}$

(d) $\left((8)^{\frac{2}{3}}\right)^{\frac{-3}{2}}$

7. Evaluate :

(a) $(.04)^{\frac{3}{2}}$

(b) $(.008)^{\frac{2}{3}}$

(c) $(.000064)^{\frac{5}{6}}$

(d) $(6.25)^{\frac{3}{2}}$

8. Evaluate :

(a) $64^{\frac{1}{2}} \times (64^{\frac{1}{2}} + 1)$

(b) $27^{-\frac{1}{3}} \times (27^{\frac{1}{3}} - 27^{\frac{2}{3}})$

(c) $4 \times 81^{-\frac{1}{2}} \times (81^{\frac{1}{2}} + 81^{\frac{3}{2}})$

(d) $\frac{36^{\frac{7}{2}} - 36^{\frac{9}{2}}}{36^{\frac{5}{2}}}$

9. Evaluate :

(a) $(3^2 + 4^2)^{\frac{1}{2}}$

(b) $(5^2 + 12^2)^{\frac{1}{2}}$

(c) $(8^2 + 15^2)^{\frac{1}{2}}$

(d) $(1^3 + 2^3 + 3^3)^{\frac{1}{2}}$

10. Simplify and express the result with positive exponents :

(a) $(x^{-4})^3$

(b) $2x^{\frac{1}{6}} \times 2x^{-\frac{7}{6}}$

(c) $x^{-7} \times y^{-7}$

(d) $x^{\frac{5}{7}} \div x^{\frac{12}{7}}$

Things to Remember

1. If q be any positive integer greater than 1, and x and y be rational numbers such that $x^q = y$, then, $y^{\frac{1}{q}} = x$.

$y^{\frac{1}{q}}$ is written as $\sqrt[q]{y}$ and read as q th root of y . $\sqrt[q]{y}$ is called a radical and q is called the index of the radical.

2. $x^{\frac{p}{q}}$ is the q th root of x^p .

3. If $\frac{p}{q}$ is a negative rational number, then $x^{\frac{p}{q}}$ ($x \neq 0$) is $\frac{1}{x^{-\frac{p}{q}}}$.

4. If x and y be any rational numbers different from zero, and a be any rational number, then

$$x^a \times y^a = (x \times y)^a.$$

5. If x be any rational number different from zero, and a and b be any rational numbers, then

$$x^a \times x^b = x^{a+b}$$

$$(x^a)^b = x^{a \times b}$$

CHAPTER FOUR

Algebraic Expressions

1 Introduction

THE PREVIOUS CLASS, we have learnt about the multiplication of algebraic expressions in the form of monomials and binomials. In particular, we have learnt about the following:

$$(1) \quad (a + b)^2 = a^2 + 2ab + b^2$$

$$(2) \quad (a - b)^2 = a^2 - 2ab + b^2$$

$$(3) \quad (a + b)(a - b) = a^2 - b^2$$

These are called identities because these are true for all values of a and b . Recall that we have used these identities in simplifying and evaluating some given expressions. We have also used these identities in the factorization of some given expressions. We shall now study some more identities and learn to apply them in simplifying, evaluating and factorizing some simple given expressions.

2 Some Identities

The Product $(x + a)(x + b)$

From identity (1), we have

$$(x + a)(x + a) = (x + a)^2 = x^2 + 2ax + a^2$$

Here, we have multiplied two identical binomials. Let us see what the product would be when the second terms of the binomials are different. In other words, let us find the product $(x + a)(x + b)$. We can find the product using the distributive property of multiplication over addition as shown below:

$$(x + a)(x + b) = x(x + b) + a(x + b)$$

$$= x^2 + xb + ax + ab$$

$$= x^2 + bx + ax + ab \quad [xb = bx]$$

$$\begin{aligned}
 &= x^2 + ax + bx + ab \quad [bx + ax = ax + bx] \\
 &= x^2 + (a + b)x + ab \quad [\text{Taking } x \text{ a common factor in } ax + bx]
 \end{aligned}$$

Thus, we have the following identity:

$$(4) \quad (x + a)(x + b) = x^2 + (a + b)x + ab$$

Remark: Note that in the R.H.S, the coefficient of x is the sum of a and b and the term independent of x is the product of a and b . Also note that a and b may take various positive or negative values. Depending on these values, we have the following identities as special cases of the above identity:

(i) When $b = a$, we have:

$$\begin{aligned}
 (x + a)(x + a) &= (x + a)^2 = x^2 + (a + a)x + a \times a \\
 &= x^2 + 2ax + a^2
 \end{aligned}$$

It is the same as identity (1).

(ii) When $b = -a$, we get

$$\begin{aligned}
 (x + a)(x - a) &= x^2 + [a + (-a)]x + a \times (-a) \\
 &= x^2 - a^2
 \end{aligned}$$

It is the same as identity (3).

$$\begin{aligned}
 \text{(iii) } (x + a)(x - b) &= x^2 + [a + (-b)]x + a \times (-b) \\
 &= x^2 + (a - b)x - ab
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } (x - a)(x - b) &= x^2 + [(-a) + (-b)]x + (-a)(-b) \\
 &= x^2 - (a + b)x + ab
 \end{aligned}$$

Let us now explain the use of this identity in simplifying and evaluating given expressions through some examples:

Example 1 : Find the following products:

$$(a) \quad (y + 3)(y + 5)$$

$$(b) \quad (p + 9)(p - 2)$$

$$(c) \quad (z - 7)(z - 5)$$

$$(d) \quad (x - 8)(x + 2)$$

Solution : (a) Let us compare $(y + 3)(y + 5)$ with $(x + a)(x + b)$. We have
 $x = y, a = 3, b = 5$

Therefore, using identity (4), we have

$$\begin{aligned}
 (y + 3)(y + 5) &= y^2 + (3 + 5)y + 3 \times 5 \\
 &= y^2 + 8y + 15
 \end{aligned}$$

Let us compare $(p + 9)(p - 2)$ with $(x + a)(x + b)$. We have

$$x = p, a = 9, b = -2$$

Therefore, using identity (4), we have

$$\begin{aligned}(p + 9)(p - 2) &= p^2 + [9 + (-2)]p + 9 \times (-2) \\ &= p^2 + 7p - 18\end{aligned}$$

Using identity (4), we have

$$\begin{aligned}(z - 7)(z - 5) &= z^2 + [(-7) + (-5)]z + (-7) \times (-5) \\ &= z^2 - 12z + 35\end{aligned}$$

Using identity (4), we have

$$\begin{aligned}(x - 8)(x + 2) &= x^2 + [(-8) + 2]x + (-8) \times 2 \\ &= x^2 - 6x - 16\end{aligned}$$

Example 2 : Evaluate the following :

(a) 107×103

(b) 56×48

Solution : (a) We can write 107 as $100 + 7$ and 103 as $100 + 3$. Thus, we have

$$\begin{aligned}107 \times 103 &= (100 + 7)(100 + 3) \\ &= (100)^2 + (7 + 3)100 + 7 \times 3 \quad [\text{Using identity (4)}] \\ &= 10000 + 10 \times 100 + 21 \\ &= 10000 + 1000 + 21 \\ &= 11021\end{aligned}$$

$$\begin{aligned}\text{(b)} \quad 56 \times 48 &= (50 + 6)(50 - 2) \\ &= (50)^2 + [6 + (-2)] \times 50 + 6 \times (-2) \quad [\text{Using identity (4)}] \\ &= 2500 + 4 \times 50 - 12 \\ &= 2500 + 200 - 12 \\ &= 2700 - 12 \\ &= 2688\end{aligned}$$

EXERCISE 4(a)

1. Find the following products :

(a) $(x + 2)(x + 9)$

(b) $(y + 7)(y + 5)$

- (c) $(x + 8)(x - 2)$
- (d) $(z - 3)(z - 5)$
- (e) $(p - 9)(p + 2)$
- (f) $(x + \frac{4}{3})(x + \frac{3}{4})$
- (g) $(y - 9)(y - 2)$
- (h) $(z^2 + 2)(z^2 - 3)$

2. Evaluate the following:

- (a) 102×106
- (b) 103×96
- (c) 95×97
- (d) 53×55
- (e) 34×36

(II) The Product $(a + b + c)^2$

Recall that identity (1) [and also identity (2)] gives an expansion for square of a binomial. We can extend this identity for the square of a trinomial and find the expansion of $(a + b + c)^2$ as follows :

Let $b + c = x$.

$$\begin{aligned}
 \therefore (a + b + c)^2 &= (a + x)^2 \\
 &= a^2 + 2ax + x^2 && \text{[Using identity (1)]} \\
 &= a^2 + 2a(b + c) + (b + c)^2 && \text{[Substituting the value of } x \text{]} \\
 &= a^2 + 2ab + 2ac + b^2 + 2bc + c^2 \\
 &= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca && \text{[Rearranging the terms]}
 \end{aligned}$$

Thus, we have the following identity :

$$(5) \quad (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

Remark: (A) Note that in the expansion for the square of $(a + b + c)$, there are 6 terms. They are:

Square of a , (a^2)

Square of b , (b^2)

Square of c , (c^2)

Twice the product of a and b , $(2ab)$

Twice the product of b and c , $(2bc)$

Twice the product of c and a , $(2ca)$

(B) You may also note that a, b, c may take various positive and negative values. Depending upon these values, we can have the following identities as special cases of identity (5) :

$$\begin{aligned} \text{i) } (a - b + c)^2 &= [a + (-b) + c]^2 \\ &= a^2 + (-b)^2 + c^2 + 2a \times (-b) + 2(-b) \times c + 2c \times a \\ &= a^2 + b^2 + c^2 - 2ab - 2bc + 2ca \end{aligned}$$

$$\begin{aligned} \text{(ii) } (a + b - c)^2 &= [a + b + (-c)]^2 \\ &= a^2 + b^2 + (-c)^2 + 2ab + 2b(-c) + 2(-c)a \\ &= a^2 + b^2 + c^2 + 2ab - 2bc - 2ca \end{aligned}$$

$$\begin{aligned} \text{iii) } (a - b - c)^2 &= [a + (-b) + (-c)]^2 \\ &= a^2 + (-b)^2 + (-c)^2 + 2a(-b) + 2(-b)(-c) + 2(-c)a \\ &= a^2 + b^2 + c^2 - 2ab + 2bc - 2ca \end{aligned}$$

$$\begin{aligned} \text{iv) } (-a + b + c)^2 &= [(-a) + b + c]^2 \\ &= (-a)^2 + b^2 + c^2 + 2(-a)b + 2bc + 2c(-a) \\ &= a^2 + b^2 + c^2 - 2ab + 2bc - 2ca, \text{ etc.} \end{aligned}$$

Let us take an example to illustrate the use of this identity in writing the square of a trinomial in expanded form.

Example 1 : Write the following in expanded form:

(a) $(9x + 2y + z)^2$

(b) $(3x - 2y - z)^2$

$$(c) (5x + y - 2z)^2$$

$$(d) (-2p + 3q + r)^2$$

Solution : (a) Comparing the given expression with $(a + b + c)^2$, we have

$$a = 9x, b = 2y, c = z$$

Therefore, using identity (5), we have:

$$\begin{aligned} (9x + 2y + z)^2 &= (9x)^2 + (2y)^2 + (z)^2 + 2(9x)(2y) + 2(2y)(z) + 2(z)(9x) \\ &= 81x^2 + 4y^2 + z^2 + 36xy + 4yz + 18zx \end{aligned}$$

(b) We can write $(3x - 2y - z)^2$ as $[3x + (-2y) + (-z)]^2$.

Comparing it with $(a + b + c)^2$, we have

$$a = 3x, b = -2y, c = -z$$

Therefore, using identity (5), we have:

$$\begin{aligned} (3x - 2y - z)^2 &= [3x + (-2y) + (-z)]^2 \\ &= (3x)^2 + (-2y)^2 + (-z)^2 + 2(3x)(-2y) + 2(-2y)(-z) + 2(-z)(3x) \\ &= 9x^2 + 4y^2 + z^2 - 12xy + 4yz - 6zx \end{aligned}$$

(c) Using identity (5), we have:

$$\begin{aligned} (5x + y - 2z)^2 &= [5x + y + (-2z)]^2 \\ &= (5x)^2 + y^2 + (-2z)^2 + 2(5x)y + 2y(-2z) + 2(-2z)(5x) \\ &= 25x^2 + y^2 + 4z^2 + 10xy - 4yz - 20zx \end{aligned}$$

(d) Using identity (5), we have:

$$\begin{aligned} (-2p + 3q + r)^2 &= (-2p)^2 + (3q)^2 + r^2 + 2(-2p)(3q) + 2(3q)r + 2r(-2p) \\ &= 4p^2 + 9q^2 + r^2 - 12pq + 6qr - 4rp \end{aligned}$$

EXERCISE 4(b)

Write the expansion of each of the following :

1. $(x + 2y + 3z)^2$

2. $(x + y - 2z)^2$

3. $(2p + 2q - 3r)^2$

4. $(p - 3q - 2z)^2$

5. $(x - 5y + 2z)^2$

6. $(9x - 2y - 3z)^2$

7. $(-3m - 5n + 2p)^2$

8. $(3x - \frac{1}{2}p + 2q)^2$

9. $(5x^2 + y + z)^2$

10. Fill in the blanks so as to make the following statements true:

(a) $(2x - 3y + 5z)^2 = \dots x^2 + \dots y^2 + \dots z^2 - \dots xy - \dots yz + \dots zx$

(b) $(x - y - 2)^2 = x^2 + y^2 + \dots y - \dots xy - \dots x + \dots$

11. Simplify:

(a) $(x + y + z)^2 + (x + y - z)^2$

(b) $(2x + p - c)^2 - (2x - p + c)^2$

12. The Product $(a + b)^3$

We can also extend identity (1) for the cube of a binomial and find $(a + b)^3$ as follows:

We know that

$$(a + b)(a + b) = (a + b)^2 = a^2 + 2ab + b^2$$

[Identity (1)]

$$\therefore (a + b)^3 = (a + b)(a + b)(a + b)$$

$$= (a + b)(a + b)^2$$

$$= (a + b)(a^2 + 2ab + b^2)$$

$$= a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2) \quad [\text{Using distributive property}]$$

$$= a \times a^2 + a \times 2ab + a \times b^2 + b \times a^2 + b \times 2ab + b \times b^2$$

$$= a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$= a^3 + 3a^2b + 3ab^2 + b^3 \quad (\text{Combining like terms})$$

$$= a^3 + b^3 + 3a^2b + 3ab^2$$

$$= a^3 + b^3 + 3ab(a + b)$$

[Taking $3ab$ a common factor in $3a^2b$ and $3ab^2$]

Thus, we have the following identity :

$$(6) \quad (a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

From the above identity, we have:

$$(a + b)^3 - 3ab(a + b) = a^3 + b^3 + 3ab(a + b) - 3ab(a + b)$$

[Subtracting $3ab(a + b)$ from both sides]

$$= a^3 + b^3$$

Thus, we have the following identity :

$$(7) \quad a^3 + b^3 = (a + b)^3 - 3ab(a + b)$$

(IV) The Product $(a - b)^3$

We can extend identity (2) to find $(a - b)^3$ as follows :

We know that

$$(a - b)(a - b) = (a - b)^2 = a^2 - 2ab + b^2$$

[Identity (2)]

$$\therefore (a - b)^3 = (a - b)(a - b)(a - b)$$

$$= (a - b)(a^2 - 2ab + b^2)$$

$$= (a - b)(a^2 - 2ab + b^2)$$

$$= a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2)$$

$$= a \times a^2 - a \times 2ab + a \times b^2 - b \times a^2 + b \times 2ab - b \times b^2$$

$$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$$

$$= a^3 - 3a^2b + 3ab^2 - b^3$$

(Combining like terms)

$$= a^3 - b^3 - 3a^2b + 3ab^2$$

$$= a^3 - b^3 - 3ab(a - b)$$

[Taking $3ab$ a common factor from $-3a^2b$ and $3ab^2$]

Thus, we have the following identity :

$$(8) \quad (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

Remarks : 1. We can also obtain the above identity from identity (6) as follows:

We have:

$$\begin{aligned}(a - b)^3 &= [a + (-b)]^3 \\ &= a^3 + (-b)^3 + 3a(-b)[a + (-b)] \quad \text{[Replacing } b \text{ by } (-b)\text{]} \\ &= a^3 - b^3 - 3ab(a - b)\end{aligned}$$

As in the case of $a^3 + b^3$, from the above identity, we can obtain the following identity:

$$(9) \quad a^3 - b^3 = (a - b)^3 + 3ab(a - b)$$

We now take some examples to illustrate the use of these identities:

Example 1 : Write the following in expanded form :

(a) $(2x + 3y)^3$

(b) $(p - yz)^3$

Solution : (a) Comparing the given expression with $(a + b)^3$, we have

$$a = 2x \text{ and } b = 3y$$

Therefore, using identity (6), we have :

$$\begin{aligned}(2x + 3y)^3 &= (2x)^3 + (3y)^3 + 3(2x)(3y)(2x + 3y) \\ &= 8x^3 + 27y^3 + 18xy(2x + 3y) \\ &= 8x^3 + 27y^3 + 36x^2y + 54xy^2 \\ &= 8x^3 + 36x^2y + 54xy^2 + 27y^3 \quad \text{[Arranging in descending order of } x\text{]}\end{aligned}$$

(b) Using the identity (8), we have:

$$\begin{aligned}(p - yz)^3 &= p^3 - (yz)^3 - 3(p)(yz)(p - yz) \\ &= p^3 - y^3z^3 - 3pyz(p - yz) \\ &= p^3 - y^3z^3 - 3p^2yz + 3py^2z^2 \\ &= p^3 - 3p^2yz + 3py^2z^2 - y^3z^3\end{aligned}$$

Example 2 : Simplify $(2x + 5)^3 - (2x - 5)^3$.

Solution : Using identity (6), we have:

$$\begin{aligned}(2x + 5)^3 &= (2x)^3 + (5)^3 + 3 \times (2x) \times 5 (2x + 5) \\ &= 8x^3 + 125 + 30x (2x + 5) \\ &= 8x^3 + 125 + 60x^2 + 150x\end{aligned}\quad (1)$$

Using identity (8), we have:

$$\begin{aligned}(2x - 5)^3 &= (2x)^3 - (5)^3 - 3 \times (2x) \times 5 (2x - 5) \\ &= 8x^3 - 125 - 30x (2x - 5) \\ &= 8x^3 - 125 - 60x^2 + 150x\end{aligned}\quad (2)$$

Subtracting (2) from (1), we obtain

$$\begin{aligned}(2x + 5)^3 - (2x - 5)^3 &= 8x^3 + 125 + 60x^2 + 150x - (8x^3 - 125 - 60x^2 + 150x) \\ &= 8x^3 + 125 + 60x^2 + 150x - 8x^3 + 125 + 60x^2 - 150x \\ &= 8x^3 - 8x^3 + 60x^2 + 60x^2 + 150x - 150x + 125 + 125 \\ &\quad \text{(Combining like terms)} \\ &= 120x^2 + 250\end{aligned}$$

Example 3 : Find the value of $a^3 + 8b^3$, if $a + 2b = 10$ and $ab = 15$.

Solution : We are given that

$$a + 2b = 10 \quad \text{and} \quad ab = 15 \quad (1)$$

Now,

$$\begin{aligned}a^3 + 8b^3 &= a^3 + (2b)^3 \\ &= (a + 2b)^3 - 3 \times a \times 2b (a + 2b) \quad \text{[Using identity (7)]} \\ &= (a + 2b)^3 - 6ab (a + 2b)\end{aligned}\quad (2)$$

Putting the values of $a + 2b$ and ab from (1) in (2), we obtain

$$\begin{aligned}a^3 + 8b^3 &= (10)^3 - 6 \times 15 \times (10) \\ &= 1000 - 900 \\ &= 100\end{aligned}$$

Example 4 : Evaluate the following:

(a) 1001^3

(b) 995^3

olution : (a) Since powers of 1000 and 1 can be written mentally, we express 1001 as $1000 + 1$ and then write

$$\begin{aligned}
 1001^3 &= (1000 + 1)^3 \\
 &= (1000)^3 + 1^3 + 3 \times 1000 \times 1 \times (1000 + 1) \quad [\text{Using identity (6)}] \\
 &= 1000000000 + 1 + 3000 \times 1001 \\
 &= 1000000000 + 1 + 3003000 \\
 &= 1000000000 + 3003001 \\
 &= 1003003001
 \end{aligned}$$

b) We express 995 as $1000 - 5$ and then write

$$\begin{aligned}
 995^3 &= (1000 - 5)^3 \\
 &= (1000)^3 - (5)^3 - 3 \times 1000 \times 5 (1000 - 5) \quad [\text{Using identity (8)}] \\
 &= 1000000000 - 125 - 15000 (995) \\
 &= 1000000000 - 125 - 14925000 \\
 &= 1000000000 - 14925125 \\
 &= 985074875
 \end{aligned}$$

EXERCISE 4(c)

1. Write the expansions of the following:

(a) $(x + 3y)^3$

(b) $(3x - 2y)^3$

(c) $(px + 2z)^3$

(d) $(x^2 + y)^3$

(e) $(2x - y^2)^3$

(f) $(7x + 9y)^3$

(g) $(2z - 7y)^3$

(h) $\left(\frac{2}{3}x - \frac{5}{3}z\right)^3$

2. Find the value of $27x^3 + 8y^3$, if

(a) $3x + 2y = 14$ and $xy = 8$

(b) $3x + 2y = 20$ and $xy = \frac{14}{9}$

3. Find the value of $p^3 - q^3$, if

(a) $p - q = -8$ and $pq = -12$

(b) $p - q = \frac{10}{9}$ and $pq = \frac{5}{3}$

4. Find the value of $64x^3 - 125z^3$, if

(a) $4x - 5z = 16$ and $xz = 12$

(b) $4x - 5z = \frac{3}{5}$ and $xz = 6$

5. Simplify:

(a) $(2x + 3p)^3 + (2x - 3p)^3$

(b) $(x + 2p)^3 - (x - 2p)^3$

(c) $(2x - z)^3 + (2x + z)^3$

(d) $(7t - 5n)^3 - (7t + 5n)^3$

(e) $\left(\frac{x}{3} + \frac{y}{5}\right)^3 - \left(\frac{x}{3} - \frac{y}{5}\right)^3$

6. Evaluate the following:

(a) 105^3 (b) 99^3 (c) 505^3 (d) 1005^3

(e) 999^3 (f) 601^3 (g) 599^3 (h) 403^3

(i) $(10.5)^3$ (j) $(9.9)^3$

7. Fill in the blanks in the following so as to make the statements true:

(a) $(mn - 3y)^3 = \dots - \dots m^2n^2y + \dots mny^2 - \dots y^3$

(b) $(x^2 + y^2)^3 = x^6 + \dots x^4y^2 + \dots y^4 + \dots$

(c) $\left(\dots - \frac{6}{7}\right)^3 = \frac{8}{27}y^3 - \dots y^2 + \dots y - \dots$

(d) $(2t^2 + 3)^3 = \dots + 27 + \dots t^4 + \dots t^2$

V) The Product $(a + b)(a^2 - ab + b^2)$

In identity (7), we have expressed $a^3 + b^3$ as $(a + b)^3 - 3ab(a + b)$. We can express $a^3 + b^3$ as a product of two expressions also. For this, let us find the product $(a + b)(a^2 - ab + b^2)$. We have :

$$\begin{aligned}
 (a + b)(a^2 - ab + b^2) &= a(a^2 - ab + b^2) + b(a^2 - ab + b^2) \\
 &= a \times a^2 + a \times (-ab) + a \times b^2 + b \times a^2 + b \times (-ab) + b \times b^2 \\
 &= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 \\
 &= a^3 - a^2b + a^2b + ab^2 - ab^2 + b^3 \quad (\text{Combining like terms}) \\
 &= a^3 + b^3
 \end{aligned}$$

Thus, we have the following identity:

$$(10) \quad a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

VI) The Product $(a - b)(a^2 + ab + b^2)$

In identity (9), we have expressed $a^3 - b^3$ as $(a - b)^3 + 3ab(a - b)$. Like $a^3 + b^3$, we can also express $a^3 - b^3$ as a product of two expressions. For this, let us find the product $(a - b)(a^2 + ab + b^2)$. We have:

$$\begin{aligned}
 (a - b)(a^2 + ab + b^2) &= a(a^2 + ab + b^2) - b(a^2 + ab + b^2) \\
 &= a \times a^2 + a \times ab + a \times b^2 - b \times a^2 - b \times ab - b \times b^2 \\
 &= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3 \\
 &= a^3 + a^2b - a^2b + ab^2 - ab^2 - b^3 \quad (\text{Combining like terms}) \\
 &= a^3 - b^3
 \end{aligned}$$

Thus, we have the following identity:

$$(11) \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Remark : Identity (11) can be directly obtained from identity (10) as follows:

$$\begin{aligned}
 a^3 - b^3 &= a^3 + (-b)^3 \\
 &= [a + (-b)][a^2 - a \times (-b) + (-b)^2] \\
 &= (a - b)(a^2 + ab + b^2)
 \end{aligned}$$

Let us take an example to illustrate the use of the above identities in finding certain products:

Example 1 : Find the following products :

$$(a) \quad (x + 3y) (x^2 - 3xy + 9y^2)$$

$$(b) \quad (25a^2 + 15ab + 9b^2) (5a - 3b)$$

Solution:

$$(a) \quad (x + 3y) (x^2 - 3xy + 9y^2)$$

$$= (x + 3y) [x^2 - x \times 3y + (3y)^2]$$

$$= x^3 + (3y)^3 \quad [\text{Using identity (10), } a = x, b = 3y]$$

$$= x^3 + 27y^3$$

$$(b) \quad (25a^2 + 15ab + 9b^2) (5a - 3b)$$

$$= (5a - 3b) (25a^2 + 15ab + 9b^2)$$

$$= (5a - 3b) [(5a)^2 + 5a \times 3b + (3b)^2]$$

$$= (5a)^3 - (3b)^3 \quad [\text{Using identity (11)}]$$

$$= 125a^3 - 27b^3$$

EXERCISE 4(d)

1. Find the following products :

$$(a) \quad (x + 2) (x^2 - 2x + 4)$$

$$(b) \quad (1 - x) (1 + x + x^2)$$

$$(c) \quad (3x + 5) (9x^2 - 15x + 25)$$

$$(d) \quad (7y - 5z) (49y^2 + 35yz + 25z^2)$$

$$(e) \quad \left(\frac{2x}{5} - \frac{3y}{7} \right) \left(\frac{9y^2}{49} + \frac{4x^2}{25} + \frac{6xy}{35} \right)$$

$$(f) \quad (0.9x + 0.7y) (0.81x^2 - 0.63xy + 0.49y^2)$$

2. Fill in the blanks in the following so as to make the statements true:

$$(a) (2t - 3) (\dots + 6t + \dots) = 8t^3 - 27$$

$$(b) (3z + 2y) (\dots - 6yz + \dots y^2) = 27z^3 + \dots y^3$$

$$(c) (x + \dots) (x^2 - \dots + 1) = x^3 + 1$$

$$(d) (x^2 + 1) (\dots - x^2 + 1) = x^6 + 1$$

3 Factorization of Algebraic Expressions

Recall that the process of writing a given number or algebraic expression as the product of two (or more) numbers or algebraic expressions is called *factorization*. The numbers or algebraic expressions that may be multiplied to obtain the given number or algebraic expression are called the *factors* of the given number or algebraic expression. For example, 3 and 5 are the factors of 15 (since $3 \times 5 = 15$). Similarly, $(x + 3)$ and $(x - 3)$ are the factors of $x^2 - 9$ [since $(x + 3)(x - 3) = x^2 - 9$]. In class VII, we have learnt to factorize some expressions utilizing the idea of greatest (highest) common factor and the three basic identities studied in that class. Here, we shall learn to factorize some more algebraic expressions utilizing the idea of greatest common factor and the identities learnt so far. Let us take some examples to illustrate the process :

Example 1 : Factorize :

$$(a) x^2 + 6x + 5$$

$$(b) x^2 - x - 12$$

$$(c) x^2 + 7x + 12$$

$$(d) x^2 + 2x - 15$$

Solution : (a) From identity (4), we know that $x^2 + (a + b)x + ab = (x + a)(x + b)$. Therefore, to factorize expressions of the type $x^2 + (a + b)x + ab$, we look for two numbers a and b such that their *sum is the coefficient of x and their product is the term independent of x* . In the case of $x^2 + 6x + 5$, the two such numbers are 5 and 1 because

$$5 + 1 = 6$$

(coefficient of x)

$$\text{and } 5 \times 1 = 5$$

(term independent of x)

Thus, we have

$$\begin{aligned}x^2 + 6x + 5 &= x^2 + (5 + 1)x + 5 \times 1 \\&= (x + 5)(x + 1) \quad \text{[Using identity (4)]}\end{aligned}$$

(b) To factorize $x^2 - x - 12$, we look for two numbers such that their sum is the coefficient of x (i.e. -1) and their product is -12 . Clearly, two such numbers are -4 and 3 because

$$\begin{aligned}-4 + 3 &= -1 \\ \text{and } (-4) \times 3 &= -12\end{aligned}$$

Thus, we have:

$$\begin{aligned}x^2 - x - 12 &= x^2 + (-4 + 3)x + (-4) \times 3 \\&= [x + (-4)][x + 3] \quad \text{[Using identity (4)]} \\&= (x - 4)(x + 3)\end{aligned}$$

[Note that 4 and -3 are not such numbers in this case.]

(c) For $x^2 - 7x + 12$, the desired numbers are -4 and -3 because

$$\begin{aligned}(-4) + (-3) &= -7 \\ \text{and } (-4) \times (-3) &= 12\end{aligned}$$

Thus, we have

$$\begin{aligned}x^2 - 7x + 12 &= x^2 + [(-4) + (-3)]x + (-4) \times (-3) \\&= [x + (-4)][x + (-3)] \quad \text{[Using identity (4)]} \\&= (x - 4)(x - 3)\end{aligned}$$

(d) For $x^2 + 2x - 15$, the desired numbers are 5 and -3 (Why?).

$$\begin{aligned}\text{Therefore, } x^2 + 2x - 15 &= x^2 + [5 + (-3)]x + 5 \times (-3) \\&= (x + 5)[x + (-3)] \\&= (x + 5)(x - 3)\end{aligned}$$

Example 2 : Factorize $x^2 + 2kx - 3k^2$.

Solution : We have:

$$\begin{aligned}x^2 + 2kx - 3k^2 &= x^2 + (3k - k)x + 3k \times (-k) \\&= (x + 3k)(x - k) \quad \text{[Using identity (4)]}\end{aligned}$$

Remark : We can also factorize the expression given in (a) of Example 1 as follows:

$$\begin{aligned}
 x^2 + 6x + 5 &= x^2 + (5 + 1)x + 5 \times 1 \\
 &= \underline{x^2 + 5x} + \underline{1 \times x + 5 \times 1} \\
 &= x(x + 5) + 1(x + 5) \\
 &= (x + 5)(x + 1) \quad \text{[Taking (x + 5) as a common factor]}
 \end{aligned}$$

The same procedure can be applied in case of other expressions.

Example 3 : Factorize the expression:

$$x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx$$

Solution : We can write the given expression as follows:

$$\begin{aligned}
 &x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx \\
 &= x^2 + (-2y)^2 + (-z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-z) + 2 \times (-z) \times (x)
 \end{aligned}$$

Let us compare the L.H.S. of the above with the L.H.S of identity (5). We have:

$$a = x, b = -2y, c = -z$$

$$\begin{aligned}
 \text{Therefore, } &x^2 + (-2y)^2 + (-z)^2 + 2 \times x \times (-2y) + 2 \times (-2y) \times (-z) + 2 \times (-z) \times (x) \\
 &= [x + (-2y) + (-z)]^2 \\
 &= (x - 2y - z)^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Thus, } x^2 + 4y^2 + z^2 - 4xy + 4yz - 2zx &= (x - 2y - z)^2 \\
 &= (x - 2y - z)(x - 2y - z)
 \end{aligned}$$

Example 4 : Factorize :

$$(a) \quad 27x^3 + 8z^3 + 54x^2z + 36xz^2$$

$$(b) \quad \frac{x^3}{8} - 64 - 3x^2 + 24x$$

Solution :

$$(a) \quad 27x^3 + 8z^3 + 54x^2z + 36xz^2$$

$$\begin{aligned}
 &= (3x)^3 + (2z)^3 + 18xz(3x + 2z) \quad \text{[Taking 18 xz a} \\
 &\hspace{15em} \text{common factor in last} \\
 &\hspace{15em} \text{two terms]}
 \end{aligned}$$

$$\begin{aligned}
 &= (3x)^3 + (2z)^3 + 3 \times (3x) \times (2z) (3x + 2z) \\
 &= (3x + 2z)^3 \quad [\text{Using identity (6); } a = 3x, b = 2z] \\
 &= (3x + 2z) (3x + 2z) (3x + 2z)
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad &\frac{x^3}{8} - 64 - 3x^2 + 24x \\
 &= \left(\frac{x}{2}\right)^3 - (4)^3 - 6x \left(\frac{x}{2} - 4\right) \\
 &= \left(\frac{x}{2}\right)^3 - (4)^3 - 3 \times \frac{x}{2} \times 4 \left(\frac{x}{2} - 4\right) \\
 &= \left(\frac{x}{2} - 4\right)^3 \quad [\text{Using identity (8)}] \\
 &= \left(\frac{x}{2} - 4\right) \left(\frac{x}{2} - 4\right) \left(\frac{x}{2} - 4\right)
 \end{aligned}$$

Example 5 : Factorize:

$$\text{(a)} \quad 1 - 8p^3$$

$$\text{(b)} \quad 128x^3 + 54z^3$$

Solution :

$$\begin{aligned}
 \text{(a)} \quad 1 - 8p^3 &= 1^3 - (2p)^3 \\
 &= (1 - 2p) [(1)^2 + 1 \times 2p + (2p)^2] \quad [\text{Using identity (1)}] \\
 &= (1 - 2p) (1 + 2p + 4p^2) \\
 \text{(b)} \quad 128x^3 + 54z^3 &= 2 (64x^3 + 27z^3) \quad [\text{Taking 2 as a common factor}] \\
 &= 2[(4x)^3 + (3z)^3] \\
 &= 2 (4x + 3z) [(4x)^2 - (4x) \times (3z) + (3z)^2] \quad [\text{Using identity (1)}] \\
 &= 2 (4x + 3z) (16x^2 - 12xz + 9z^2)
 \end{aligned}$$

Example 6 : Factorize $x^6 - y^6$.

Solution : Remember that x and y represent numbers and, therefore, all the rules or laws of exponents for numbers are valid for them also. Thus, we have:

$$\begin{aligned}
 x^6 - y^6 &= (x^3)^2 - (y^3)^2 \quad [(x^m)^n = x^{mn}] \\
 &= (x^3 + y^3) (x^3 - y^3) \quad [\text{Using identity (3)}]
 \end{aligned}$$

that each of the expressions $(x^3 + y^3)$ and $(x^3 - y^3)$ can be further factorized. Factorizing $x^3 + y^3$ and $x^3 - y^3$ using identities (10) and (11) respectively, we have:

$$\begin{aligned} x^6 - y^6 &= (x^3 + y^3)(x^3 - y^3) \\ &= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2) \end{aligned}$$

Other Method :

$$\begin{aligned} x^6 - y^6 &= (x^2)^3 - (y^2)^3 \\ &= (x^2 - y^2)[(x^2)^2 + x^2y^2 + (y^2)^2] \quad [\text{Using identity (11); } a = x^2, b = y^2] \\ &= (x + y)(x - y)(x^4 + x^2y^2 + y^4) \\ &= (x + y)(x - y)(x^4 + 2x^2y^2 + y^4 - x^2y^2) \\ &= (x + y)(x - y)[(x^2 + y^2)^2 - (xy)^2] \\ &= (x + y)(x - y)(x^2 + y^2 + xy)(x^2 + y^2 - xy) \end{aligned}$$

Remark: Recall that an expression is said to be completely factorized, if none of the factors can be further factorized. In general, we use the word 'factorize' in the sense of completely factorizing a given expression.

EXERCISE 4 (e)

1. Factorize the following expressions:

(a) $x^2 + 9x + 20$

(b) $x^2 - 6x + 8$

(c) $z^2 + z - 12$

(d) $p^2 + 5pq - 36q^2$

(e) $m^2 - 2m - 15$

(f) $m^2 + 11mn + 18n^2$

2. Factorize the following expressions:

(a) $x^2 + 4y^2 + z^2 + 4xy - 2xz - 4yz$

(b) $4p^2 + 9q^2 + 4r^2 + 12pq + 12qr + 8pr$

(c) $4x^2 + y^2 + 9z^2 - 4xy + 6yz - 12zx$

(d) $p^2 + pq + \frac{q^2}{4} + 1 + 2p + q$

(e) $2x^2 + 4y^2 + 3z^2 + 4\sqrt{2}xy - 4\sqrt{3}yz - 2\sqrt{6}xz$

[Hint : $2x^2 + 4y^2 + 3z^2 = (\sqrt{2}x)^2 + (2y)^2 + (\sqrt{3}z)^2$, etc.]

3. Factorize the following expressions:

(a) $27x^3 - 8y^3 - 54x^2y + 36xy^2$

(b) $x^3 + 64y^3 + 12x^2y + 48xy^2$

(c) $8y^3 - 125z^3 - 60y^2z + 150yz^2$

(d) $x^3 + 125z^3 + 75xz^2 + 15x^2z$

4. Factorize the following expressions:

(a) $p^3 + 27$

(b) $y^3 + 125$

(c) $1 - 27z^3$

(d) $8x^3y^3 + 27z^3$

(e) $64x^3 - y^3$

(f) $m^3 - 27n^3$

(g) $\frac{1}{216} p^3 - 8q^3$

5. Factorize the following expressions:

(a) $10xy^4 - 10x^4y$

(b) $54x^6y + 2x^3y^4$

(c) $32p^3 + 108q^3$

(d) $p^6 - 64q^6$

(e) $(p - 2q)^3 - (8q)^3$

[Hint : $p^6 - 64q^6 = (p^3)^2 - (8q^3)^2$]

(f) $27(x - 1)^3 + m^3$

[Hint : $27(x - 1)^3 + m^3 = \{3(x - 1)\}^3 + m^3$, etc.]

4.4 One More Identity

We have factorized $a^3 + b^3$. One very natural question is : Can we always factorize $a^3 + b^3 + c^3$? If not, can we factorize it in some special cases? To answer the question, let us consider the product

$$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Using the distributive property, we have:

$$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a(a^2 + b^2 + c^2 - ab - bc - ca) + b(a^2 + b^2 + c^2 - ab - bc - ca) + c(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= a^3 + ab^2 + ac^2 - a^2b - abc - a^2c + a^2b + b^3 + bc^2 - ab^2 - b^2c - a^2c + b^2c + c^3 - abc - bc^2 - ac^2$$

$$= a^3 + b^3 + c^3 + ab^2 - ab^2 + ac^2 - ac^2 - a^2b + a^2b - abc - abc - abc - a^2c + a^2c + bc^2 - bc^2 - b^2c + b^2c$$

(Combining like terms)

$$= a^3 + b^3 + c^3 - 3abc$$

Thus, we have the following identity :

$$(12) \quad a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

Now, let us assume that a, b, c have such values that $a + b + c = 0$. For example,

$$a = 2, b = -3, c = 1.$$

Then, R.H.S. of identity (12) will be

$$0 \times (a^2 + b^2 + c^2 - ab - bc - ca) = 0 \quad [\text{Since } a + b + c = 0]$$

Therefore, in the above situation,

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$\text{or, } a^3 + b^3 + c^3 = 3abc$$

Thus, we have the following result:

$$\text{If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc$$

Note that the result $a^3 + b^3 + c^3 = 3abc$ is not true for all values of a, b, c . It is only true when the values of a, b, c satisfy the condition $a + b + c = 0$. For this reason, the result (A) is called a *conditional identity*. We say that: A *conditional identity is an equation in two or more unknowns which is true only when the unknowns satisfy a certain condition*.

We are now in a state to answer the question of factorizing $a^3 + b^3 + c^3$. Note that under the condition $a + b + c = 0$, we can express $a^3 + b^3 + c^3$ as $3abc$. In other words, under the condition $a + b + c = 0$, we can factorize $a^3 + b^3 + c^3$ as $3abc$.

Remark : If instead of $a + b + c = 0$, we assume that $a^2 + b^2 + c^2 - ab - bc - ca = 0$ (for example, take $a = 1, b = 1, c = 1$), then we can obtain the following conditional identity :

$$\text{If } a^2 + b^2 + c^2 - ab - bc - ca = 0, \text{ then}$$

$$a^3 + b^3 + c^3 = 3abc$$

Let us illustrate the use of these identities through some examples .

Example 1 : Factorize the expression :

$$8x^3 + 27y^3 - z^3 + 18xyz$$

Solution : We have

$$\begin{aligned} & 8x^3 + 27y^3 - z^3 + 18xyz \\ &= (2x)^3 + (3y)^3 + (-z)^3 - 3(2x)(3y)(-z) \\ &= [2x + 3y + (-z)] [(2x)^2 + (3y)^2 + (-z)^2 - (2x)(3y) - (3y)(-z) - (2x)(-z)] \\ & \quad \quad \quad \text{[Using identity (12); } a = 2x, b = 3y, c = -z\text{]} \\ &= (2x + 3y - z)(4x^2 + 9y^2 + z^2 - 6xy + 3yz + 2zx) \end{aligned}$$

Example 2 : Factorize:

$$(x - y)^3 + (y - z)^3 + (z - x)^3$$

Solution : Let us compare $(x - y)^3 + (y - z)^3 + (z - x)^3$ with the L.H.S. of the conditional identity (A).

We have:

$$a = x - y, b = y - z \text{ and } c = z - x$$

$$\text{Therefore, } a + b + c = x - y + y - z + z - x = 0$$

Hence, we can write

$$\begin{aligned} (x - y)^3 + (y - z)^3 + (z - x)^3 &= 3(x - y)(y - z)(z - x) \\ & \quad \quad \quad \text{[Using conditional identity (A)]} \end{aligned}$$

Example 3 : Find the value of $45^3 - 65^3 + 20^3$.

Solution : We can write

$$45^3 - 65^3 + 20^3 \text{ as } (45)^3 + (-65)^3 + (20)^3$$

$$\text{We note that } 45 + (-65) + 20 = 0$$

Therefore, using the conditional identity (A), we have:

$$\begin{aligned} 45^3 - 65^3 + 20^3 &= (45)^3 + (-65)^3 + (20)^3 \\ &= 3 \times 45 \times (-65) \times 20 \\ &= -175500 \end{aligned}$$

EXERCISE 4 (f)

1. Factorize the following expressions:

(a) $p^3 + 8q^3 + 64r^3 - 24pqr$

(b) $8x^3 - 27y^3 + z^3 + 18xyz$

(c) $l^3 + m^3 - n^3 + 3lmn$

(d) $-27x^3 + y^3 - z^3 - 9xyz$

(e) $x^3 - 8y^3 - 64z^3 - 24xyz$

(f) $\frac{1}{27} x^3 - y^3 + 125z^3 + 5xyz$

2. Factorize the following expressions :

(a) $(3x - 5y)^3 + (5y - 9z)^3 + (9z - 3x)^3$

(b) $(p-3q)^3 + (3q-7r)^3 + (7r-p)^3$

(c) $(5x - 6p)^3 + (7z - 5x)^3 + (6p - 7z)^3$

(d) $(\frac{1}{2} x - 3y)^3 + (3y - \sqrt{3} z)^3 + (\sqrt{3} z - \frac{1}{2} x)^3$

(e) $p^3 (q - r)^3 + q^3 (r - p)^3 + r^3 (p - q)^3$

[Hint : $p^3 (q - r)^3 = [p (q - r)]^3 = (pq - rp)^3$, etc.]

3. Find the value of :

(a) $55^3 - 25^3 - 30^3$

(b) $47^3 + 29^3 - 76^3$

(c) $(9.8)^3 - (11.3)^3 + (1.5)^3$

(d) $(2.7)^3 - (1.6)^3 - (1.1)^3$

Things to Remember

1. (i) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(ii) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(iii) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(iv) $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)(a^2 - ab + b^2)$

(v) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(vi) $a^3 - b^3 = (a - b)^3 + 3ab(a - b) = (a - b)(a^2 + ab + b^2)$

(vii) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

(2.) If $(a + b + c) = 0$, then $a^3 + b^3 + c^3 = 3abc$

CHAPTER FIVE

Division of Algebraic Expressions

1 Introduction

WE ARE ALREADY familiar with division of numbers. Recall that when we want to divide 18 by 6, we ask ourselves the question : "By what number should we multiply 6 to obtain 18 ?" We note that the answer to this question is 3.

Thus, we say that $18 \div 6 = 3$

$$\text{or, } \frac{18}{6} = 3$$

Division of algebraic expressions is also defined in a similar manner. For example, let us consider the two monomials $4x^2y$ and $3xy^2$. We can easily see that

$$(4x^2y) \times (3xy^2) = 12x^3y^3 \quad (1)$$

We can, therefore, say that

$$12x^3y^3 \div (3xy^2) = 4x^2y \quad (2)$$

In this case $12x^3y^3$ is called the *dividend*, $3xy^2$ is called the *divisor* and $4x^2y$ is called the *quotient*. As in the case of numbers, we can also write above as

$$\frac{12x^3y^3}{3xy^2} = 4x^2y \quad (3)$$

Remarks :

(i) From (1), we can also say that

$$12x^3y^3 \div 4x^2y = 3xy^2$$

and write it in the following form :

$$\frac{12x^3y^3}{4x^2y} = 3xy^2 \quad (4)$$

(ii) As x, y represent numbers, they follow all the laws of exponents that are valid for numbers. You can verify this fact from (3) and (4) above.

(iii) Division by zero is not defined.

5.2 Some Rational Expressions and Their Simplification

Let us now consider the two expressions $y^2 + 8y + 15$ and $y + 3$. We know from earlier Chapter Section 4.2 that

$$(y + 3)(y + 5) = y^2 + 8y + 15$$

We can, therefore, say that

$$(y^2 + 8y + 15) \div (y + 3) = y + 5$$

As usual, $y^2 + 8y + 15$ is the dividend, $y + 3$ is the divisor and $y + 5$ is the quotient.

Also, we can write the above as

$$\frac{y^2 + 8y + 15}{y + 3} = y + 5$$

Similarly, we can write that

$$\frac{x^2 + (a + b)x + ab}{x + a} = x + b \quad [\text{Since } (x + a)(x + b) = x^2 + (a + b)x + ab]$$

$$\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b \quad [\text{Why ?}]$$

$$\frac{a^3 - b^3}{a - b} = a^2 + ab + b^2, \text{ etc.}$$

The above results can also be obtained as follows :

$$\frac{y^2 + 8y + 15}{y + 3} = \frac{(y + 3)(y + 5)}{(y + 3)} = (y + 5) \quad [\text{Just as in numbers, we cancel common factor } (y + 3) \text{ in numerator and denominator}]$$

$$\frac{a^3 + b^3}{a^2 - ab + b^2} = \frac{(a + b)(a^2 - ab + b^2)}{a^2 - ab + b^2} = (a + b) \quad [\text{Cancelling common factor } (a^2 - ab + b^2)]$$

$$\frac{a^3 - b^3}{a - b} = \frac{(a - b)(a^2 + ab + b^2)}{a - b} = a^2 + ab + b^2, \text{ etc.}$$

Expressions of the type $12x^3y^3$, $3xy^2$, $y^2 + 8y + 15$, $x^2 + (a + b)x + ab$, $x + a$, $+ b^3$, $a^2 - ab + b^2$, etc. in which the variables involved have only non-negative integral exponents are called *polynomials* and the expressions of the type

$\frac{y^2 + 8y + 15}{y + 3}$, $\frac{a^3 + b^3}{a^2 - ab + b^2}$, etc. are called *rational expressions*. Note that so far,

we had been discussing about algebraic expressions which, in fact, were polynomials. As you can easily observe from the above results that sometimes we can divide a polynomial by another polynomial and obtain some other polynomial. In other words, on simplifying certain rational expressions, we may obtain polynomials in the end. It can also be seen that the various identities learnt by us help us in simplifying these expressions. We shall take some more examples to illustrate this process :

Example 1 : Divide $8a^3 - 27b^3$ by $2a - 3b$.

Solution : $(8a^3 - 27b^3) \div (2a - 3b)$

$$= \frac{8a^3 - 27b^3}{2a - 3b}$$

$$= \frac{(2a)^3 - (3b)^3}{(2a - 3b)}$$

$$= \frac{(2a - 3b)[(2a)^2 + 2a \times 3b + (3b)^2]}{(2a - 3b)} \quad \begin{array}{l} \text{[Factorizing } 8a^3 - 27b^3, \text{ identity} \\ \text{(11)]} \end{array}$$

$$= \frac{(2a - 3b)(4a^2 + 6ab + 9b^2)}{(2a - 3b)}$$

$$= 4a^2 + 6ab + 9b^2 \quad \begin{array}{l} \text{[Cancelling the common} \\ \text{factor } (2a - 3b) \text{ in numerator} \\ \text{and denominator]} \end{array}$$

Example 2 : Simplify :

$$(a) \frac{64y^3 - 1000}{8y - 20}$$

$$(b) \frac{x^2 + 7x + 12}{x + 3}$$

Solution :

$$(a) \frac{64y^3 - 1000}{8y - 20}$$

$$= \frac{(4y)^3 - (10)^3}{2(4y - 10)}$$

$$= \frac{(4y - 10)(16y^2 + 40y + 100)}{2(4y - 10)}$$

[Identity (11)]

$$= \frac{16y^2 + 40y + 100}{2}$$

[Cancelling $(4y - 10)$]

$$= \frac{4(4y^2 + 10y + 25)}{2}$$

$$= 2(4y^2 + 10y + 25)$$

(Cancelling 2)

$$= 8y^2 + 20y + 50$$

$$(b) \frac{x^2 + 7x + 12}{x + 3} = \frac{x^2 + (3 + 4)x + 12}{x + 3}$$

$$= \frac{(x + 3)(x + 4)}{(x + 3)}$$

[Identity (4)]

$$= x + 4$$

[Cancelling $x + 3$]

EXERCISE 5 (a)

1. Divide :

(a) $6x^2yz$ by $3xy$

(b) $15m^2n^3$ by $5m^2n^2$

(c) $x^2 - y^2$ by $x + y$

(d) $27x^3 + y^3$ by $9x^2 - 3xy + y^2$

2. Simplify :

(a) $\frac{16m^3y^2}{4m^2y}$

(b) $\frac{x^2 + 4x + 4}{x + 2}$

$$(c) \frac{16m^2 - 9n^2}{4m - 3n}$$

$$(d) \frac{125x^3 + 64}{25x^2 - 20x + 16}$$

$$(e) \frac{9x^2 - 24xy + 16y^2}{3x - 4y}$$

$$(f) \frac{216z^3 - 343p^3}{6z - 7p}$$

3 Polynomials in One Variable and Their Division

As already stated in Section 5.2, the expressions $12x^3y^3$, $3xy^2$, $y^2 + 8y + 15$, $-ab + b^2$, etc. are all polynomials. The expressions of the type $2x^4 - 4x^2 + 12$, $6m^2 + 3mn - 10m + 5n$, etc., which we came across in Class VII, were also polynomials. The polynomial $-6m^2 + 3mn - 10m + 5n$ involves two variables m and n . It is called a *polynomial in two variables m and n* . The polynomial

$2x^4 - 4x^2 + 12$ involves only one variable x . Therefore, it is called a *polynomial in one variable x* or simply a *polynomial in x* . Similarly, $y^2 + 8y + 15$ is a polynomial in y while $3xy^2$ is a polynomial in variables x and y . Note that the highest exponent of the variable involved in various terms of $2x^4 - 4x^2 + 12$ is 4. We say that the polynomial $2x^4 - 4x^2 + 12$ is of *degree 4*. Also, the degree of the term $2x^4$ is 4, of $4x^2$ is 2 and that of 12 is 0 (since we can write 12 as $12x^0$). Similarly, the polynomial $y^2 + 8y + 15$ is of *degree 2*. In general, the highest exponent of the variable in various terms of the polynomial in one variable is called its *degree*. For example, degree of the polynomial $5z^3 - 2z^5 + 7z^2 + 8$ is 5, degree of polynomial $-8 + \frac{9}{2}y^3$ is 3, degree of $x^6 - x^5 + x^7 - \sqrt{2}x^2$ is 7, etc.

Remarks : (i) In the case of polynomials in more than one variable, we find the sum of the exponents of the variables involved in each term and the highest sum obtained is called the *degree* of the polynomial. For example, in the polynomial $x^4 - 3x^3y^2 + 3x^2y^2 + y^3$, the sum of the exponents of the variables in

$$x^4 \text{ is } 4 + 0 = 4,$$

$$-3x^3y^2 \text{ is } 3 + 2 = 5,$$

$$3x^2y^2 \text{ is } 2 + 2 = 4,$$

$$\text{and } y^3 \text{ is } 0 + 3 = 3$$

Therefore, the degree of the polynomial is 5.

(ii) As $x^0 = 1$, the degree of the polynomial 7 (i.e. $7x^0$) is zero.

We shall now discuss the division of a monomial by a monomial and utilize the knowledge in the division of a polynomial in one variable by a monomial or binomial (in one variable). We shall explain the process through some examples.

(I) Division of a Monomial by a Monomial

Let us consider the division of the monomial $10x^3y^2$ by the monomial $-5xy$.
 $(-5xy) \times (-2x^2y) = 10x^3y^2$, we can say that $10x^3y^2 \div (-5xy) = -2x^2y$ i.e.,

$$\frac{10x^3y^2}{-5xy} = -2x^2y$$

Note that $\frac{10}{-5} = -2$ and $\frac{x^3y^2}{xy} = x^2y$

Similarly, we can see that

$$\frac{-15x^4}{-3x^2} = 5x^2$$

$$\left[\frac{-15}{-3} = 5 \text{ and } \frac{x^4}{x^2} = x^2 \right]$$

$$\frac{21x^5}{-3x^3} = -7x^2$$

$$\left[\frac{21}{-3} = -7 \text{ and } \frac{x^5}{x^3} = x^2 \right]$$

Thus, while dividing a monomial by a monomial, we follow the two rules given below :

1. The coefficient of the quotient of two monomials is equal to the quotient of the coefficients.
2. The variable part in the quotient of two monomials is equal to the quotient of the variables in the given monomials.

(II) Division of a Polynomial by a Monomial

Suppose we want to divide a polynomial $4x^2 + 6x^3 - 28x$ by the monomial $2x$. perform the division as follows :

Step 1 : We arrange the terms of the dividend in descending order of their degree. Thus, we write $4x^2 + 6x^3 - 28x$ as

$$6x^3 + 4x^2 - 28x$$

Step 2 : We divide each term of the polynomial by the given monomial $2x$, using the above mentioned two rules.

us, we obtain that

$$\begin{aligned}(6x^3 + 4x^2 - 28x) \div 2x &= \frac{6x^3}{2x} + \frac{4x^2}{2x} - \frac{28x}{2x} \\ &= 3x^2 + 2x - 14\end{aligned}$$

b) Division of a Polynomial by a Binomial

Suppose we want to divide a polynomial $-12 - 6p^2 + 17p$ by a binomial $2p - 3$.

We perform the division as follows :

Step 1 : We arrange the terms of the dividend and divisor in descending order of their degrees. Thus, we write $-12 - 6p^2 + 17p$ as $-6p^2 + 17p - 12$ and $2p - 3$ as $2p - 3$.

Step 2 : We divide the first term of the dividend by the first term of the divisor. Thus, we divide $-6p^2$ by $2p$ and obtain $-3p$. $-3p$ is the first term of the quotient.

Step 3 : We multiply the divisor by the first term of the quotient and subtract the result from the dividend. Thus, we multiply $2p - 3$ by $-3p$ and subtract their product $-6p^2 + 9p$ from $-6p^2 + 17p - 12$. We obtain $8p - 12$ as the remainder.

Step 4 : We treat $8p - 12$ as the new dividend and repeat Step 2 to obtain the second term of the quotient, i.e., we divide $8p$ by $2p$ and obtain $+4$ as the second term of the quotient.

Step 5 : We multiply the divisor $2p - 3$ by the second term of the quotient namely, $+4$ and subtract the result from the new dividend. Thus, we subtract the product of $(2p - 3)$ and $+4$ from $8p - 12$, i.e., we subtract $8p - 12$ from $8p - 12$. We obtain 0 as the remainder.

Hence, we say that

$$(-6p^2 + 17p - 12) \div (2p - 3) = -3p + 4$$

The above process can be displayed as shown below :

$$\begin{array}{r} -3p + 4 \\ 2p - 3 \overline{) -6p^2 + 17p - 12} \\ \underline{-6p^2 + 9p} \\ 8p - 12 \\ \underline{8p - 12} \\ 0 \end{array}$$



Thus, in this case, the quotient is $-3p + 4$ and the remainder is 0.

Note that in the above two cases, the remainder is zero. We say that $2x$ is a *factor* of the polynomial $4x^2 + 6x^3 - 28x$ and $2p - 3$ is a *factor* of the polynomial $-12 - 6p^2 + 17p$. In general, if on dividing a polynomial (dividend) by another polynomial (divisor), we obtain a zero remainder, then the second polynomial (divisor) is called a *factor* of the first polynomial (dividend). What can we say about the quotient? Is it also a factor of the dividend? We know that if on dividing a number by another number we obtain a zero remainder, then we can write :

$$\text{Dividend} = \text{Divisor} \times \text{Quotient}$$

In the same way, if on dividing a polynomial by another polynomial we obtain a zero remainder, then we can write :

$$\text{Dividend} = \text{Divisor} \times \text{Quotient}$$

In other words, we can say that *in such a case, the quotient is also a factor of the dividend*. Thus, $3x^2 + 2x - 14$ is a factor of $6x^3 + 4x^2 - 28x$ and $-3p + 4$ is a factor of $-6p^2 + 17p - 12$. Let us take one more example to illustrate the above process.

Example 1 : Divide the polynomial $2y^4 + 7y^2 + 8y^3 + 4y + 3$ by $3 + y$. Is $y + 3$ a factor of $2y^4 + 7y^2 + 8y^3 + 4y + 3$?

Solution : We go through the following steps to perform the division :

Step 1 : We write the terms of the dividend as well as of the divisor in descending order of their degrees. Thus, we write $2y^4 + 7y^2 + 8y^3 + 4y + 3$ as $2y^4 + 8y^3 + 7y^2 + 4y + 3$ and $3 + y$ as $y + 3$.

Step 2 : We divide the first term ($2y^4$) of the dividend by the first term (y) of the divisor and obtain $2y^3$ as the first term of the quotient.

Step 3 : We multiply the divisor ($y + 3$) by the first term ($2y^3$) of the quotient and subtract the result ($2y^4 + 6y^3$) from the dividend $2y^4 + 8y^3 + 7y^2 + 4y + 3$. We obtain $2y^3 + 7y^2 + 4y + 3$ as the remainder.

Step 4 : We treat $2y^3 + 7y^2 + 4y + 3$ as the new dividend and repeat Step 2 to obtain the second term of the quotient, i.e., we divide $2y^3$ by y and obtain $2y^2$ as the second term of the quotient.

Step 5 : We multiply the divisor ($y + 3$) by the second term ($2y^2$) of the quotient and subtract the result ($2y^3 + 6y^2$) from the new dividend. We obtain $y^2 + 4y + 3$.

Step 6 : Now we treat $y^2 + 4y + 3$ as the new dividend and divide its first term (y^2) by the first term (y) of the divisor to obtain the third term of the quotient as y .

Step 7 : We multiply the divisor $(y + 3)$ by the third term (y) of the quotient and subtract the result $(y^2 + 3y)$ from the new dividend. We obtain $y + 3$ as the remainder.

Step 8 : Now we treat this remainder $y + 3$ as the new dividend and divide its first term (y) by the first term (y) of the divisor to obtain the fourth term (1) of the quotient.

Step 9 : We multiply the divisor $(y + 3)$ by the fourth term of the quotient (1) and subtract the result $(y + 3)$ from $(y + 3)$. We obtain 0 as the remainder.

Thus, we can say that

$$(2y^4 + 8y^3 + 7y^2 + 4y + 3) \div (y + 3) = 2y^3 + 2y^2 + y + 1$$

The above process can be displayed as follows :

$$\begin{array}{r}
 2y^3 + 2y^2 + y + 1 \\
 y + 3 \overline{) 2y^4 + 8y^3 + 7y^2 + 4y + 3} \\
 \underline{- 2y^4 + 6y^3} \\
 2y^3 + 7y^2 + 4y + 3 \\
 \underline{- 2y^3 + 6y^2} \\
 y^2 + 4y + 3 \\
 \underline{- y^2 + 3y} \\
 y + 3 \\
 \underline{- y + 3} \\
 0
 \end{array}$$

As the remainder is zero, we can say that $y + 3$ is a factor of the given polynomial $2y^4 + 8y^3 + 7y^2 + 4y + 3$.

V) Division with a Non-zero Remainder

In the examples of (I), (II) and (III) above, we have seen that the remainder in each case is zero. In such cases (as in the case of numbers), we say that *the dividend is completely divisible or exactly divisible or simply divisible by the divisor*. Now we consider the case in which the remainder is not zero. Recall that in the case of numbers, we continue the division till we obtain a remainder which is less than the divisor. In the case of polynomials, we continue the division till we obtain a

remainder whose degree is less than that of the divisor. We shall illustrate it through examples.

Example 2 : Divide $9x^3 + 3x^2 + 7 - 5x$ by $3x - 1$.

Solution : We go through the following steps to perform the division :

Step 1 : We write the terms of dividend and also of the divisor in descending order of their degrees. Thus, we write $9x^3 + 3x^2 + 7 - 5x$ as $9x^3 + 3x^2 - 5x + 7$ and $3x - 1$ as $3x - 1$.

Step 2 : We divide the first term ($9x^3$) of the dividend by the first term ($3x$) of the divisor and obtain $3x^2$ as the first term of the quotient.

Step 3 : We multiply the divisor ($3x - 1$) by the first term ($3x^2$) of the quotient and subtract the result ($9x^3 - 3x^2$) from the dividend ($9x^3 + 3x^2 - 5x + 7$). We obtain $6x^2 - 5x + 7$.

Step 4 : We treat $6x^2 - 5x + 7$ as the new dividend and repeat Step 2 to obtain the second term of the quotient, i.e., we divide $6x^2$ by $3x$ and obtain $2x$ as the second term of the quotient.

Step 5 : We multiply the divisor ($3x - 1$) by the second term ($2x$) of the quotient and subtract the result ($6x^2 - 2x$) from the new dividend $6x^2 - 5x + 7$. We obtain $-3x + 7$ as the remainder.

Step 6 : Now, we treat $-3x + 7$ as the new dividend and repeat Step 2 to obtain the third term of the quotient, i.e. we divide $(-3x)$ by $3x$ and obtain -1 as the third term of the quotient.

Step 7 : We multiply the divisor ($3x - 1$) by the third term (-1) of the quotient and subtract the result $-3x + 1$ from the new dividend $-3x + 7$. We obtain 6 as the remainder.

Note that the degree of remainder 6 is 0 and that of divisor $3x - 1$ is 1 . The degree of 6 is less than the degree of $3x - 1$. Therefore, we stop the division process and write $3x^2 + 2x - 1$ as the quotient and 6 as the remainder.

DIVISION OF ALGEBRAIC EXPRESSIONS

The above process can be displayed as follows :

$$\begin{array}{r}
 3x^2 + 2x - 1 \\
 3x - 1 \overline{) 9x^3 + 3x^2 - 5x + 7} \\
 \underline{- 9x^3 \quad + 3x^2} \\
 6x^2 - 5x + 7 \\
 \underline{- 6x^2 \quad + 2x} \\
 -3x + 7 \\
 \underline{- 3x + 1} \\
 6
 \end{array}$$

Thus, Quotient = $3x^2 + 2x - 1$

Remainder = 6

In the case of numbers, the relation

Dividend = Divisor $\times$ Quotient + Remainder

is true for division of polynomials also. We can check the correctness of our division using this relation as follows :

Check : Divisor $\times$ Quotient + Remainder

$$\begin{aligned}
 &= (3x - 1)(3x^2 + 2x - 1) + 6 \\
 &= 3x(3x^2 + 2x - 1) - 1(3x^2 + 2x - 1) + 6 \\
 &= 9x^3 + 6x^2 - 3x - 3x^2 - 2x + 1 + 6 \\
 &= 9x^3 + 3x^2 - 5x + 7 = \text{Dividend}
 \end{aligned}$$

Example 3 : Divide the polynomial $6x^5 + 4x^4 - 27x^3 - 7x^2 - 27x - 6$ by the polynomial $2x^2 - 3$.

Solution : We proceed in the same manner as in Example 2. The division process

can be displayed as follows :

$$\begin{array}{r}
 3x^3 + 2x^2 - 9x - \frac{1}{2} \\
 2x^2 - 3 \overline{) 6x^5 + 4x^4 - 27x^3 - 7x^2 - 27x - 6} \\
 \underline{- 6x^5} \qquad \qquad \underline{- 9x^3} \\
 4x^4 - 18x^3 - 7x^2 - 27x - 6 \\
 \underline{- 4x^4} \qquad \qquad \underline{+ 6x^2} \\
 - 18x^3 - x^2 - 27x - 6 \\
 \underline{+ 18x^3} \qquad \underline{+ 27x} \\
 - x^2 - 54x - 6 \\
 \underline{+ x^2} \qquad \underline{+ \frac{3}{2}} \\
 - 54x - \frac{15}{2}
 \end{array}$$

Note that the division process ends when we obtain the remainder $-54x - \frac{15}{2}$. It is because the degree of $-54x - \frac{15}{2}$ is 1 and that of the divisor $2x^2 - 3$ is 2.

Thus, Quotient $= 3x^3 + 2x^2 - 9x - \frac{1}{2}$

Remainder $= -54x - \frac{15}{2}$

Check : Divisor $\times$ Quotient + Remainder

$$\begin{aligned}
 &= (2x^2 - 3) \left(3x^3 + 2x^2 - 9x - \frac{1}{2} \right) + \left(-54x - \frac{15}{2} \right) \\
 &= 2x^2 \left(3x^3 + 2x^2 - 9x - \frac{1}{2} \right) - 3 \left(3x^3 + 2x^2 - 9x - \frac{1}{2} \right) \\
 &\quad + \left(-54x - \frac{15}{2} \right) \\
 &= 6x^5 + 4x^4 - 18x^3 - x^2 - 9x^3 - 6x^2 + 27x + \frac{3}{2} - 54x - \frac{15}{2} \\
 &= 6x^5 + 4x^4 - 18x^3 - 9x^3 - x^2 - 6x^2 + 27x - 54x + \frac{3}{2} - \frac{15}{2} \\
 &= 6x^5 + 4x^4 - 27x^3 - 7x^2 - 27x - 6 \\
 &= \text{Dividend}
 \end{aligned}$$

Remarks : (i) Terms of the dividend as well as of the divisor should be arranged in descending order of their degrees.

- (ii) Degree of the remainder is always less than that of the divisor.
- (iii) In general, we think of division of polynomials only when the degree of the divisor is less than that of the dividend.
- (iv) When the remainder is not zero, then the divisor (or quotient) is not a factor of the dividend.

EXERCISE 5 (b)

1. Write the degree of each of the following polynomials :

- (a) $2x^3 + 5x^2 - 7$ (b) $5x^2 - 3x + 2 + 3x^4$
- (c) $2x + x^2 - 8$ (d) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$
- (e) $5x^2$ (f) $20x^3 + 12x^2y^2 - 10y^2 + 20$
- (g) 6

2. Divide :

- (a) $9m^5 + 12m^4 - 6m^2$ by $3m^2$
- (b) $x^2 + 7x + 12$ by $x + 3$
- (c) $4y^2 + 3y + \frac{1}{2}$ by $2y + 1$
- (d) $4z^3 + 6z^2 - z$ by $-\frac{1}{2}z$
- (e) $3m^3 + 4m^2 + 5m + 18$ by $m + 2$
- (f) $3y^4 - 3y^3 - 4y^2 - 4y$ by $y^2 - 2y$
- (g) $2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3$ by $2y^3 + 1$

3. Divide :

- (a) $14x^2 + 13x - 15$ by $7x - 4$
- (b) $15z^3 - 20z^2 + 13z - 12$ by $3z - 6$
- (c) $6y^5 - 28y^3 + 3y^2 + 30y - 9$ by $2y^2 - 6$
- (d) $34x - 22x^3 - 12x^4 - 10x^2 - 75$ by $3x + 7$
- (e) $15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6$ by $3y - 2$

In each case, check your answer using the relation

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}.$$

4. Using division state whether

- (a) $x + 6$ is a factor of $x^2 - x - 42$.
- (b) $4z - 3$ is a factor of $4z^2 - 13z - 12$.
- (c) $2y - 5$ is a factor of $4y^4 - 10y^3 - 10y^2 + 30y - 15$.
- (d) $3y^2 + 5$ is a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$.
- (e) $z^2 + 3$ is a factor of $z^5 - 9z$.

Things to Remember

1. Expressions in which the variables involved have only non-negative integral exponents are called polynomials and the expressions of the type $\frac{y^2 + 8y + 15}{y + 3}$ are called rational expressions.
2. The highest exponent of the variable in various terms of a polynomial in one variable is called its degree. In the case of polynomials in more than one variable, we find the sum of the exponents of the variables involved in each term and the highest sum so obtained is called the degree of the polynomial.
3. Degree of any constant term is zero.
4. The coefficient of the quotient of two monomials is equal to the quotient of their coefficients.
5. The variable part in the quotient of two monomials is equal to the quotient of the variables in the given monomials.
6. If on dividing a polynomial (dividend) by another polynomial (divisor), a zero remainder is obtained, then the divisor is called a factor of the dividend. In such cases, quotient is also a factor of the dividend. Further,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient}$$

In such a case, we say that the dividend is completely divisible or

exactly divisible or simply divisible by the divisor.

7. In general,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

8. For dividing a polynomial by another polynomial, the terms of the dividend as well as of the divisor should be arranged in descending order of their degrees.
9. The degree of the remainder is always less than that of the divisor.

CHAPTER SIX

Equations in One Variable

6.1 Introduction

IN OUR EARLIER CLASSES, we have learnt to solve linear equations in one variable and solved some word problems by converting them into the form of equations. Can you recall the rules that we have used to solve those equations? The rules are that, to solve an equation, we can

- I. Add the same number on both sides of the equation.
- II. Subtract the same number from both sides of the equation.
- III. Multiply both sides of the equation by the non-zero same number.
- IV. Divide both sides of the equation by the same non-zero number.

You may also recall that on the basis of the above rules I and II, we can transpose a term from one side of the equation to the other side after changing its sign (from + to - and from - to +).

Now, we shall learn to solve some equations in one variable which can be converted into the form of linear equations in one variable. We shall also solve some more word problems by first converting them into the form of equations in one variable.

6.2 Equation of the form $\frac{ax + b}{cx + d} = k$

Let us consider an equation $\frac{3x + 5}{2x + 7} = 4$.

It is an equation of the form $\frac{ax + b}{cx + d} = k$, where $a = 3$, $b = 5$, $c = 2$, $d = 7$ and $k = 4$. Note that it is an equation in one variable but it is not linear (Why?). Similar to the equations

$\frac{5x - 7}{3x} = 2$, $\frac{-4x + 3}{2x + 6} = -3$, etc. are of the type $\frac{ax + b}{cx + d} = k$, where a, b, c, d are

are some constants and $cx + d \neq 0$. They are not linear equations in one variable. However, we can solve these type of equations by converting them into the form of linear equations. We shall illustrate the process through examples :

Example 1 : Solve the equation $\frac{5x-7}{3x} = 2$.

Solution : We have :

$$\frac{5x-7}{3x} = 2 \quad (1)$$

As x represents a number, so $3x$ also represents a number. Therefore, multiplying both sides of equation (1) by $3x$, we have

$$\frac{(5x-7)}{3x} \times 3x = 2 \times 3x \quad (\text{Rule III})$$

$$\text{or, } (5x-7) \times 1 = 6x \quad \left(\frac{3x}{3x} = 1 \right)$$

$$\text{or, } 5x-7 = 6x$$

$$\text{or, } 5x-6x = 7 \quad [\text{Transposing the terms } 6x \text{ and } -7]$$

$$\text{or, } -x = 7$$

$$\text{or, } \frac{-x}{-1} = \frac{7}{-1} \quad (\text{Rule IV})$$

$$\text{or, } x = -7$$

Thus, $x = -7$ is the solution of the given equation.

$$\begin{aligned} \text{Check : At } x = -7, \text{ L.H.S.} &= \frac{5(-7)-7}{3(-7)} = \frac{-35-7}{-21} = \frac{-42}{-21} = 2 \\ &= \text{R.H.S.} \end{aligned}$$

Hence, our solution is correct.

Example 2 : Solve the equation $\frac{-4x+3}{2x+6} = -3$.

Solution : We have :

$$\frac{-4x+3}{2x+6} = -3$$

As x represents a number, therefore $2x + 6$ also represents a number.

Therefore, multiplying both sides of the equation by $2x + 6$, we have :

$$\frac{(-4x + 3)}{(2x + 6)} \times (2x + 6) = -3 \times (2x + 6) \quad (\text{Rule III})$$

$$\text{or,} \quad (-4x + 3) \times 1 = -6x - 18 \quad \left[\frac{2x + 6}{2x + 6} = 1 \right]$$

$$\text{or,} \quad -4x + 3 = -6x - 18$$

$$\text{or,} \quad -4x + 6x = -18 - 3 \quad [\text{Transposing the terms } -6x \text{ and } 3]$$

$$\text{or,} \quad 2x = -21$$

$$\text{or,} \quad \frac{2x}{2} = \frac{-21}{2} \quad (\text{Rule IV})$$

$$\text{or,} \quad x = -\frac{21}{2}$$

Thus, $x = -\frac{21}{2}$ is the solution of the given equation.

$$\text{Check : At } x = -\frac{21}{2},$$

$$\text{L.H.S} = \frac{-4x + 3}{2x + 6}$$

$$= \frac{-4 \left(\frac{-21}{2} \right) + 3}{2 \left(\frac{-21}{2} \right) + 6}$$

$$= \frac{42 + 3}{-21 + 6} = \frac{45}{-15}$$

$$= -3 = \text{R.H.S.}$$

Hence, our solution is correct.

Example 3 : Solve the equation $\frac{2-y}{y+7} = \frac{3}{5}$.

Solution : We have :

$$\frac{2-y}{y+7} = \frac{3}{5}$$

Multiplying both sides of the equation by $y + 7$, we have :

$$\frac{(2-y)}{(y+7)} \times (y+7) = \frac{3}{5} \times (y+7) \quad (\text{Rule III})$$

$$2-y = \frac{3y+21}{5}$$

Now multiplying both sides of the equation by 5, we have

$$(2-y) \times 5 = \frac{(3y+21)}{5} \times 5 \quad (\text{Rule III})$$

$$10-5y = 3y+21 \quad \left[\frac{5}{5} = 1 \right]$$

$$-5y-3y = 21-10 \quad [\text{Transposing the terms } 10 \text{ and } 3y]$$

$$-8y = 11$$

$$\frac{-8y}{-8} = \frac{11}{-8} \quad (\text{Rule IV})$$

$$y = -\frac{11}{8}$$

Thus, $y = -\frac{11}{8}$ is the solution of the given equation.

Check: At $y = -\frac{11}{8}$,

$$\text{L.H.S.} = \frac{2 - \left(-\frac{11}{8}\right)}{-\frac{11}{8} + 7}$$

$$= \frac{\frac{16+11}{8}}{-\frac{11+56}{8}}$$

$$= \frac{\frac{27}{8}}{\frac{45}{8}} = \frac{27}{8} \times \frac{8}{45} = \frac{27}{45}$$

$$= \frac{3}{5} = \text{R.H.S.}$$

Thus, our solution is correct.

Remark: In brief, the method of solving the above equation can be written as follows

$$\frac{2-y}{y+7} = \frac{3}{5}$$

$$\text{or, } (2-y) \times 5 = 3 \times (y+7)$$

$$\text{or, } 10 - 5y = 3y + 21$$

$$\text{or, } -5y - 3y = 21 - 10$$

$$\text{or, } -8y = 11$$

$$\text{or, } y = -\frac{11}{8}$$

This method is popularly known as the method of *cross multiplication*.

Example 4 : Solve the equation

$$\frac{2x-3}{3x+2} = \frac{-2}{3}$$

Solution : We have

$$\frac{2x-3}{3x+2} = \frac{-2}{3}$$

Multiplying both sides of the equation by $3x+2$, we have

$$\frac{(2x-3)}{(3x+2)} \times (3x+2) = \frac{-2}{3} \times (3x+2) \quad (\text{Rule III})$$

$$\text{or, } 2x-3 = \frac{-2(3x+2)}{3}$$

$$\text{or, } (2x-3) \times 3 = -2(3x+2) \quad (\text{Rule III})$$

$$\text{or, } 6x-9 = -6x-4$$

$$6x + 6x = -4 + 9$$

[Transposing -9 and $-6x$]

$$12x = 5$$

$$\frac{12x}{12} = \frac{5}{12}$$

$$x = \frac{5}{12}$$

ck: At $x = \frac{5}{12}$,

$$\text{L.H.S.} = \frac{2 \times \frac{5}{12} - 3}{3 \times \frac{5}{12} + 2}$$

$$= \frac{\frac{5}{6} - 3}{\frac{5}{4} + 2}$$

$$= \frac{\frac{5 - 18}{6}}{\frac{5 + 8}{4}}$$

$$= \frac{-\frac{13}{6}}{\frac{13}{4}}$$

$$= -\frac{13}{6} \times \frac{4}{13}$$

$$= \frac{-2}{3}$$

$$= \text{R.H.S.}$$

e, our solution is correct.

Remark: By cross multiplication, the above equation can be solved as follows

$$\frac{2x-3}{3x+2} = \frac{-2}{3}$$

or, $3 \times (2x-3) = -2 \times (3x+2)$

or, $6x-9 = -6x-4$

or, $6x+6x = -4+9$ [Transposing -9 and $-$

or, $12x = 5$

or, $x = \frac{5}{12}$

EXERCISE 6 (a)

Solve the following equations and check your solutions :

1. $\frac{3x+5}{2x+7} = 4$ $(3x+5) = 4(2x+7)$

2. $\frac{2y+5}{y+4} = 1$ $3x+5 = 8x+28$

3. $\frac{2x+1}{3x-2} = \frac{5}{9}$ $3x-8 = 2x-5$
 $-5x = 23$

4. $\frac{5z-3}{2z} = \frac{8}{9}$ $5z = \frac{-5}{23} - \left(\frac{23}{-5}\right)$

5. $\frac{1-9y}{19-3y} = \frac{5}{8}$

$N = 3 \left(\frac{23}{-5} \right) 5$

$69 \times 25 = 1725$ $\frac{69}{-5} \times 25 = -69$

$\frac{3+23}{69}$

$\frac{4}{69+5}$
 $\frac{345}{69}$

Solve the following equations:

$$6. \quad \frac{0.4z-3}{1.5z+9} = \frac{-7}{5}$$

$$7. \quad \frac{5y-3}{2y+1} = \frac{2}{5}$$

$$8. \quad \frac{3y+5}{3-2y} = \frac{5}{3}$$

$$9. \quad \frac{2x}{3x+1} = -3$$

$$10. \quad \frac{17(2-x)-5(x+12)}{1-7x} = 8$$

$$11. \quad \frac{y-(7-8y)}{9y-(3+4y)} = \frac{2}{3}$$

Applications of Equations to Solve Problems

In our earlier classes, we have already solved some real life problems with the help of equations in one variable. Recall that the important steps in the solution of these problems are :

1. We read the problem carefully and note down what is given and what is required.
2. We denote the unknown by some letter say x , y , z , etc.
3. We translate the statements of the problem step by step into mathematical statements, to the extent it is possible.
4. We look for the quantities which are equal and write the equation by writing appropriate expressions for these quantities.
5. We solve the equation for the unknown.
6. We check whether the obtained solution satisfies the conditions given in the problem.

Now, we consider some more problems at a higher level than that of earlier classes:

Example 1 : The difference of two positive numbers is 72 and the quotient obtained on dividing the one by the other is 4. Find the numbers.

Solution : Let one of the numbers be x .

Since the difference of the numbers is 72, therefore the other number will be $x + 72$.

According to the condition of the problem,

$$\frac{72 + x}{x} = 4 \quad \text{[Their quotient is given as 4]}$$

$$\text{or,} \quad \left(\frac{72 + x}{x} \right) \times x = 4 \times x \quad \text{[Rule III]}$$

$$\text{or,} \quad 72 + x = 4x$$

$$\text{or,} \quad x - 4x = -72 \quad \text{[Transposing 72 and 4x]}$$

$$\text{or,} \quad -3x = -72$$

$$\text{or,} \quad \frac{-3x}{-3} = \frac{-72}{-3} \quad \text{[Rule IV]}$$

$$\text{or,} \quad x = 24$$

Thus, one number is 24 and the other number is $24 + 72 = 96$.

Check: (1) Difference of the two numbers $= 96 - 24 = 72$, which is the same as given in the problem.

(2) Quotient of the numbers $= \frac{96}{24} = 4$, which is the same as given in the problem.

Thus, our solution is correct.

Remark: In (1), we cannot take $\frac{x}{72 + x} = 4$, because the numbers are positive and hence $72 + x$ is always greater than x .

Example 2 : An alloy of silver and gold weighs 30g in air and 28g in water. Assuming that silver loses one-tenth of its weight in water and that gold loses one-nineteenth of its weight; find the weight of each metal in the alloy.

Solution : Let the weight of silver in the alloy be x g.

Therefore, weight of gold in the alloy = $(30 - x)g$

Now weight lost by silver in water = $\frac{1}{10} \times x \times g$

$$\begin{aligned} \text{Weight of silver in water} &= \left(x - \frac{x}{10} \right) g \\ &= \left(\frac{10x - x}{10} \right) g = \frac{9x}{10} g \end{aligned} \quad (1)$$

Weight lost by gold in water = $\frac{1}{19} \times (30 - x) g$

$$\begin{aligned} \text{Weight of gold in water} &= \left[(30 - x) - \frac{1}{19} \times (30 - x) \right] g \\ &= \left[\frac{(30 - x) \times 19 - (30 - x)}{19} \right] g \\ &= \left(\frac{570 - 19x - 30 + x}{19} \right) g \\ &= \left(\frac{540 - 18x}{19} \right) g \end{aligned} \quad (2)$$

Weight of the alloy in water

$$= \left[\frac{9x}{10} + \frac{540 - 18x}{19} \right] g \quad [\text{From (1) and (2)}]$$

$$= \left[\frac{9x \times 19 + 10(540 - 18x)}{190} \right] g$$

$$= \left[\frac{171x + 5400 - 180x}{190} \right] g$$

$$= \left[\frac{5400 - 9x}{190} \right] g$$

Weight of the alloy in water is given as 28 g.

$$\text{Therefore, } \frac{5400 - 9x}{190} = 28$$

$$\text{or, } \frac{5400 - 9x}{190} \times 190 = 28 \times 190$$

$$\text{or, } 5400 - 9x = 5320$$

$$\text{or, } -9x = 5320 - 5400$$

$$\text{or, } -9x = -80$$

$$\text{or, } 9x = 80$$

$$\text{or, } x = \frac{80}{9} = 8\frac{8}{9}$$

Thus, weight of silver in the alloy is $8\frac{8}{9}$ g and weight of gold in the alloy is:

$$\left(30 - \frac{80}{9}\right) \text{ g} = \left(\frac{270 - 80}{9}\right) \text{ g}$$

$$= \frac{190}{9} \text{ g} = 21\frac{1}{9} \text{ g}$$

Check: (1) Weight of the alloy in air = $8\frac{8}{9} \text{ g} + 21\frac{1}{9} \text{ g} = 30 \text{ g}$, which is the same as given in the problem.

$$\begin{aligned} \text{(2) Weight of silver in water} &= \left(\frac{80}{9} - \frac{1}{10} \times \frac{80}{9}\right) \text{ g} \\ &= \frac{72}{9} \text{ g} = 8 \text{ g} \end{aligned}$$

$$\text{Weight of gold in water} = \left(\frac{190}{9} - \frac{1}{19} \times \frac{190}{9}\right) \text{ g}$$

$$= \left(\frac{190}{9} - \frac{10}{9}\right) \text{ g}$$

$$= \frac{180}{9} \text{ g} = 20 \text{ g}$$

Therefore, weight of the alloy in water = $8\text{g} + 20\text{g} = 28\text{g}$, which is the same as given in the problem.

Example 3 : The digit in the tens place of a two-digit number is three times that in the ones place. If the digits are reversed, the new number will be 36 less than the original number. Find the number.

Solution : Let the digit in the ones place be x .

Therefore, the digit in the tens place = $3x$.

Note that if the digit at the ones place is 3 and that at tens place is 2, then the number will be $10 \times 2 + 3 = 20 + 3 = 23$. Similarly, if the digit at the ones place be 5 and that at the tens place is 3, then the number = $10 \times 3 + 5 = 35$, etc.

In the same way, the number in this case = $10 \times (\text{digit at tens place}) + \text{digit at ones place} = 10 \times (3x) + x = 30x + x = 31x$.

If the digits are reversed, the digit at the ones place will be $3x$ and that at tens place will be x . Then, the new number will be

$$10 \times x + 3x = 10x + 3x = 13x$$

It is given that the above number is less than the original number by 36.

Therefore, $31x - 36 = 13x$

or, $31x - 13x = 36$

(Transposing $13x$ and -36)

or, $18x = 36$

or, $x = \frac{36}{18} = 2$

Thus, ones digit is 2 and tens digit is $3 \times 2 = 6$.

Hence, the required number is 62.

Check: (1) 3 times digit at ones place = $3 \times 2 = 6 =$ digit at the tens place, which is the same as given in the problem.

On reversing the digits, the new number = 26. Also, $62 - 26 = 36$, which is the same as given in the problem.

Example 4 : The denominator of a rational number is greater than its numerator by 4. If the numerator is increased by 11 and the denominator is decreased by 1, the new number becomes $\frac{7}{3}$. Find the original number.

Solution : Let the numerator be x .

$\therefore$ denominator = $x + 4$

Thus, the given rational number = $\frac{x}{x+4}$ (1)

Now, in the second case,

$$\text{numerator} = x + 11$$

$$\text{and denominator} = x + 4 - 1 = x + 3$$

$$\text{Thus, the new number} = \frac{x + 11}{x + 3}$$

According to the condition of the problem, the new number is equal to $\frac{7}{3}$.

$$\text{Therefore, } \frac{x + 11}{x + 3} = \frac{7}{3}$$

$$\text{or, } 3 \times (x + 11) = 7 \times (x + 3) \quad (\text{By cross multiplication})$$

$$\text{or, } 3x + 33 = 7x + 21$$

$$\text{or, } 3x - 7x = 21 - 33 \quad (\text{Transposing } 7x \text{ and } 33)$$

$$\text{or, } -4x = -12$$

$$\text{or, } \frac{-4x}{-4} = \frac{-12}{-4} \quad (\text{Rule IV})$$

$$\text{or, } x = 3$$

$$\begin{aligned} \text{Therefore, original number} &= \frac{x}{x + 4} = \frac{3}{3 + 4} \\ &= \frac{3}{7} \quad [\text{From (1)}] \end{aligned}$$

Check : (1) $7 - 3 = 4$ i.e. denominator is greater than numerator by 4, which is given in the problem.

(2) New numerator $= 3 + 11 = 14$ and new denominator $= 7 - 1 = 6$.

Hence, new number $= \frac{14}{6} = \frac{7}{3}$, which is the same as given in the problem.

Example 5: The distance between two stations A and B is 230 km. Two motor cyclists start simultaneously from A and B in the opposite directions and the distance between them after three hours is 20 km. If the speed of one motor cyclist is less than that of the other by 10 km/hr, find the speed of each motor cyclist.

Solution : Let the speed of the slower motor cyclist be x km/hr and let he starts from A.

The speed of the other motor cyclist = $(x + 10)$ km/hr
 The distance AC travelled by the first motor cyclist in

$$\begin{aligned} 3 \text{ hours} &= x \times 3 \text{ km} \\ &= 3x \text{ km} \end{aligned}$$



Fig 6.1

The distance BD travelled by the other motor cyclist in 3 hours = $(x + 10) \times 3$ km
 $= (3x + 30)$ km

It is given that the distance CD between them after 3 hours is 20 km (See Fig. 6.1).
 Now, we know that

$$AC + CD + BD = 230 \text{ km}$$

$$3x + 20 + 3x + 30 = 230$$

$$6x + 50 = 230$$

$$6x = 230 - 50$$

$$6x = 180$$

$$x = \frac{180}{6}$$

$$x = 30$$

Thus, speed of the first motor cyclist = 30 km/hr and that of the second motor cyclist
 $(30 + 10)$ km/hr = 40 km/hr.

Check: Distance AC travelled by first motor cyclist in 3 hours = 30×3 km = 90 km
 Distance BD travelled by the second motor cyclist in 3 hours = 40×3 km = 120 km
 Therefore, distance between them after 3 hours

$$= [230 - (90 + 120)] \text{ km}$$

$$= (230 - 210) \text{ km}$$

$$= 20 \text{ km, which is the same as given in the problem.}$$

Example 6: The ages (in years) of Ramesh and Rahim are in the ratio 5 : 7. If Ramesh
 be 9 years older and Rahim 9 years younger, the age of Ramesh would have been
 twice the age of Rahim. Find their ages.

Solution: Let age (in years) of Ramesh be $5x$.

Therefore, age (in years) of Rahim = $7x$ (Ratio of their ages is 5 : 7)

Had Ramesh be 9 years older, then his age would have been $(5x + 9)$ years.

Had Rahim be 9 years younger, then his age would have been $(7x - 9)$ years.

According to the condition of the problem, we have

$$(5x + 9) = 2(7x - 9)$$

$$\text{or, } 5x + 9 = 14x - 18$$

$$\text{or, } 5x - 14x = -18 - 9 \quad (\text{Transposing 9 and 14x})$$

$$\text{or, } -9x = -27$$

$$\text{or, } x = \frac{-27}{-9} = 3$$

Thus, age of Ramesh = 5×3 years = 15 years and that of Rahim = 7×3 years = 21 years.

Check : (1) Ratio of their ages is $15 : 21 = 5 : 7$, which is the same as given in problem.

(2) If Ramesh were 9 years older, his age = $(15 + 9)$ years = 24 years

If Rahim were 9 years younger, his age = $(21 - 9)$ years = 12 years

24 years is twice of 12 years, which is the same as given in the problem.

EXERCISE 6 (b)

1. The sum of two numbers is 45 and their ratio is 7 : 8. Find the numbers.
2. The sum of the digits of a two digit number is 12. If the new number formed by reversing the digits is greater than the original number by 18, find the original number. Check your solution.
3. The distance between two stations is 340 km. Two trains start simultaneously from these stations on parallel tracks to cross each other. The speed of one of them is greater than that of the other by 5 km/hr. If the distance between the two trains after 2 hours of their start is 30 km, find the speed of each train. Check your solution.
4. A steamer goes downstream and covers the distance between two ports in 4 hours while it covers the same distance upstream in 5 hours. If the speed of the stream is 2 km per hour, find the speed of the steamer in still water.

[Hint : Let speed of the steamer in still water be x km/hr.

Speed downstream = $(x + 2)$ km/hr,

Speed upstream = $(x - 2)$ km/hr, etc.]

5. The numerator of a rational number is less than its denominator by 3. If the numerator becomes three times and the denominator is increased by 20, the new number becomes $\frac{1}{8}$. Find the original number.
6. Meera's mother is four times as old as Meera. After five years, her mother will be three times as old as she will be then. What are their present ages ?
7. The digit at the tens place of a two digit number is three times the digit at the ones place. If the sum of this number and the number formed by reversing its digits is 88, find the number.
8. The length of a rectangle exceeds its breadth by 4 cm. If length and breadth are each increased by 3 cm, the area of the new rectangle will be 81 sq.cm more than that of the given rectangle. Find the length and breadth of the given rectangle. Check your solution.
9. An altitude of a triangle is five-thirds the length of its corresponding base. If the altitude were increased by 4 cm and the base decreased by 2 cm, the area of the triangle would remain the same. Find the base and altitude of the triangle.
10. The sum of the weights of an iron piece and of a copper piece is 1280 g. The volume of the copper piece is twice that of the iron piece. If the weight of 1 cu.cm of iron is 7.8 g and that of copper is 8.9 g, find the volume of each piece.
11. A field can be ploughed in 14 days. If 20 hectares of land is ploughed more daily, the work will be finished in 10 days. Find the area of the land which can be ploughed daily. Also, find the area of field.
12. The sum of two numbers is 2490. If 6.5% of one number is equal to 8.5% of the other, find the numbers.
13. Three prizes are to be distributed in a quiz contest. The value of the 2nd prize is five-sixths the value of the first prize and the value of the third prize is four-fifths that of the 2nd prize. If the total value of the three prizes is Rs 150, find the value of each prize.
14. One of the angles of a triangle is equal to the sum of the other two angles. If the ratio of the other two angles is 4 : 5, find the angles of the triangle.

Things to Remember

1. For solving an equation, we can transpose a term from one side to the other after changing its sign (from $+$ to $-$ and from $-$ to $+$).
2. Method of cross multiplication can also be used in solving some equations.

CHAPTER SEVEN

Compound Interest

THIS CHAPTER we shall discuss the concept of compound interest and the method of calculating the amount and the compound interest. We shall also discuss growth of population, depreciation of value of assets and other real life situations wherein the method of calculating compound interest is applicable.

Compound Interest

In Class VII, you learnt how to calculate simple interest if principal, rate and time are known. If P denotes the principal (in Rs), $R\%$ per annum is the rate and T denotes time (in years), then the simple interest (in Rs) is given by the formula

$$\text{S.I.} = \frac{P \times R \times T}{100}$$

For example, if principal is Rs 2000 and the rate is 10% per annum, then the simple interest for two years will be

$$\text{Rs } \frac{2000 \times 10 \times 2}{100} = \text{Rs } 400$$

However, in practice, the method according to which banks, insurance corporations and other money lending and deposit taking companies calculate interest, is not exactly the one given above. To understand the way they calculate interest, we consider an example :

Suppose Deenu, a small farmer, borrows from Regional Rural Bank a loan worth Rs 1000 to purchase good quality seeds. If the bank charges interest at the rate of 10%

per annum, then at the end of one year, Deenu owes to the bank

(a) borrowed amount (the principal) = Rs 2000 and

$$\begin{aligned} \text{(b) interest on Rs 2000 for one year} &= \text{Rs } \frac{2000 \times 10 \times 1}{100} \\ &= \text{Rs 200} \end{aligned}$$

Thus, a total sum of Rs 2200 is what Deenu has to pay to the bank at the end of first year. If for some reason (bad crop etc.), Deenu is unable to pay the interest (the principal) at the end of the first year, the bank will charge from Deenu interest on Rs 2200 thereafter. Thus, the principal for the second year would become Rs 2200 which as a matter of fact, is the amount (Rs 2000 + Rs 200) at the end of the first year. At the end of the second year, Deenu owes to the bank

(a) new principal = Rs 2200 and

$$\begin{aligned} \text{(b) interest on Rs 2200 for one year} &= \text{Rs } \frac{2200 \times 10 \times 1}{100} \\ &= \text{Rs 220} \end{aligned}$$

Thus, the total interest payable to the bank at the end of the second year is Rs (200 + 220) = Rs 420. This interest is Rs 20 more than the simple interest on Rs 2000 for 2 years at the same rate. As you must have noticed, this excess in interest is due to the fact that the principal for the second year was more than the principal for the first year. The interest calculated in this manner is called *compound interest*. The meaning of compound interest will become more clear after going through some illustrative examples :

Example 1 : Find the compound interest on Rs 1000 for two years at 4% per annum.

Solution : Principal for the first year = Rs 1000

$$\text{Rate (per cent per annum)} = 4$$

$$\begin{aligned} \text{Interest for the first year} &= \text{Rs } \frac{1000 \times 4 \times 1}{100} \\ &= \text{Rs 40} \end{aligned}$$

$$\begin{aligned} \therefore \text{Amount at the end of the first year} &= \text{Rs } (1000 + 40) \\ &= \text{Rs 1040} \end{aligned}$$

$$\text{Principal for the second year} = \text{Rs 1040}$$

$$\text{Interest for the second year} = \text{Rs } \frac{1040 \times 4 \times 1}{100}$$

$$= \text{Rs } 41.60$$

efore, the compound interest at the end of the second year = Interest for the first
+ interest for the second year = Rs (40 + 41.60) = Rs 81.60

ark : (a) The simple interest in the above example would be Rs 80 only.
(b) The interest for the second year exceeds the interest for the first year
by Rs 1.60. This excess is, infact, the interest on Rs 40 (the interest for
the first year).

e above illustration, we multiplied the principal for a particular year by the rate
ent and divided the product by 100 to get the interest for that year. In decimal
ion, division by 100 means multiplication by .01 which is equivalent to shifting
ecimal point to the left by two digits.

ustrate the use of decimal notation in the following example.

ple 2 : Anita borrowed a sum of Rs 2000 from a nationalized bank to purchase
ting machine. If the rate of interest is 5% per annum, calculate the compound.
est that Anita has to pay to the bank after three years.

ion : Principal for the first year = Rs 2000

Interest for the first year = Rs 100

Multiply by 5 and divide by 100

Amount at the end of the first year

or

Add

Principal for the second year = Rs 2100

Interest for the second year = Rs 105

Multiply by 5 and shift deci-
mal point two places left

Amount at the end of the second year

or the principal for the third year = Rs 2205

Add

Interest for the third year = Rs 110.25

Multiply by 5 and shift
decimal point two places left

Compound interest at the end of the third year = Rs (100 + 105 + 110.25)
= Rs 315.25

ark: The compound interest at the end of the third year can also be calculated
by calculating the amount at the end of the third year (Rs 2315.25 in the present
and then by subtracting from it the principal at the beginning of the first year
(2000)).

Let us now summarize the main differences between simple interest and compound interest.

The main difference between simple interest and compound interest is that in simple interest the principal remains constant throughout whereas in the case of compound interest it goes on changing every year. The principal for a particular year (from second year onwards) is the sum of the principal for the previous year and the compound interest for that year. For a given principal, rate and time, the compound interest is generally more than the simple interest.

The compound interest for the entire period can be calculated either by adding the compound interest for all the years or by subtracting from the final amount the original principal.

EXERCISE 7 (a)

1. Find the compound interest when principal = Rs 3000, rate = 5% per annum and time = 2 years.
2. What will be the compound interest on a principal of Rs 5000 in 2 years when the rate of interest is 10% per annum?
3. Salama borrowed from *Mahila Samiti* a sum of Rs 625 to purchase a sewing machine. If the rate of interest is 4% per annum, what is the compound interest that she has to pay after two years?
4. Dheerubhai borrowed from District Cooperative Bank a sum of Rs 2000 to instal a handpump in his dairy. If the rate of interest is 4% per annum, calculate the compound interest that Dheerubhai pays to the bank after 3 years.
5. Vasudevan deposited Rs 8000 with a finance company for 3 years at an interest of 15% per annum. What is the compound interest that Vasudevan gets after 3 years?
6. Daljeet, an unemployed youth, received Rs 4000 as a loan at a *loan mela* organized by the State Bank of India. If the nominal interest is $2\frac{1}{2}$ % per annum, calculate the compound interest that Daljeet pays after two years.
7. Calculate the compound interest that Gokul Chand will get by investing Rs 1000 for 3 years at the rate of 10% per annum.

3. To renovate his shop, Arif obtained a loan of Rs 8000 from Allahabad Bank. If the rate of interest be 5% per annum, calculate the compound interest that Arif pays to the bank after 3 years.
9. John purchased a black and white TV set on credit. If the set costs Rs 2400 and the shopkeeper charges interest at the rate of 20% per annum, calculate the compound interest that John will have to pay after 3 years.
10. Swati took a loan of Rs 16000 against her insurance policy at the rate of $12\frac{1}{2}$ % per annum. Calculate the total compound interest payable by Swati after 3 years.

Formula for Finding the Compound Interest

Method of calculating compound interest discussed in the preceding Section is tedious and cumbersome, specially when the period of time is large. In this Section, we obtain a formula for calculating compound interest. To derive the formula, we review the calculations done in Examples 1 and 2 discussed in the previous Section.

In Example 1, the interest for the first year is Rs $1000 \times 4 \times .01$ and the amount at the end of the first year is Rs $(1000 + 1000 \times 4 \times .01)$, i.e., Rs $1000(1 + 4 \times .01)$. The amount is actually Rs 1040, but we write it in the present form for the purpose of finding some pattern and hence a formula.

The interest for the second year is Rs $1040 \times 4 \times .01$ and, therefore, the amount at the end of the second year is Rs $(1040 + 1040 \times 4 \times .01)$, i.e., Rs $1040(1 + 4 \times .01)$. Writing $1000(1 + 4 \times .01)$ for 1040, we get the amount at the end of the second year as Rs $1000(1 + 4 \times .01)(1 + 4 \times .01)$, i.e., Rs $1000(1 + 4 \times .01)^2$. If we write P for the principal, which is Rs 1000, R % for the rate per annum which is 4%, and n for the number of years which is 2 in this case, the amount A (in Rs) at the end of the n th year can be written as

$$A = P(1 + R \times .01)^n$$

In the second example, $P = 2000$, $R = 5$ and $n = 3$. The interest for the first year is Rs $2000 \times 5 \times .01$ and, therefore, the amount (in Rs) at the end of the first year is Rs $2000 + 2000 \times 5 \times .01 = 2000(1 + 5 \times .01) = 2000(1 + 5 \times .01)^1$. The interest for the second year is $2000(1 + 5 \times .01) \times 5 \times .01$ and, therefore, the amount at the end of the second year is $2000(1 + 5 \times .01) + 2000(1 + 5 \times .01) \times 5 \times .01 = 2000(1 + 5 \times .01)(1 + 5 \times .01) = 2000(1 + 5 \times .01)^2$. Using this amount as the

principal for the third year, the amount at the end of the third year can be calculated as $2000 (1 + 5 \times .01)^3$. This amount A in terms of P , R and n can be written as

$$A = P(1 + R \times .01)^n.$$

From the above illustrations, it follows that if P denotes the principal (in Rs), R denotes the rate per annum, and n denotes the number of years, then the amount A (in Rs) at the end of the n th year is given by the formula

$$A = P(1 + R \times .01)^n$$

Once the amount A is known, the compound interest, C.I., can be obtained by subtracting P from A . Since

$$\begin{aligned} A - P &= P(1 + R \times .01)^n - P \\ &= P \{(1 + R \times .01)^n - 1\}, \end{aligned}$$

the compound interest can also be calculated by using the formula

$$\text{C.I.} = P \{(1 + R \times .01)^n - 1\}$$

Remark: In the formula for A (and C.I.), R denotes the interest on Rs 100 for one year. Therefore, $R \times .01$ is the interest on Re 1 for one year. Thus, if we denote the rate of interest per rupee per annum by r , then the formula for the amount will become

$$A = P(1 + r)^n, \text{ where } r = R \times .01$$

Consequently, the compound interest will be given by

$$\text{C.I.} = P \{(1 + r)^n - 1\}$$

Example 1 : Find the amount of Rs 4000 for two years at 5% per annum compound interest.

Solution : Here P (in Rs) = 4000, $R = 5$ and $n = 2$. Therefore, the amount A (in Rs) is given by

$$\begin{aligned} A(\text{in rupees}) &= P(1 + R \times .01)^n \\ &= 4000 (1 + 5 \times .01)^2 \\ &= 4000 \times 1.05 \times 1.05 \\ &= 4410 \end{aligned}$$

Example 2 : Vijay obtains a loan of Rs 64000 against his fixed deposits. If the rate of interest be 2.5 paise per rupee per annum, calculate the compound interest payable after 3 years.

Solution : Here $P = \text{Rs } 64000$, $n = 3$ years, and $r = 2.5 \text{ paise} = .025 \text{ rupees (per rupee per annum)}$. Hence, the compound interest C.I. (in Rs) is given by

$$\begin{aligned} \text{C.I. (in rupees)} &= P \{(1 + r)^n - 1\} \\ &= 64000 \{(1 + .025)^3 - 1\} \\ &= 4921 \end{aligned}$$

EXERCISE 7 (b)

In problems 1- 5 , calculate the amount, by using the formula for compound interest :

1. Principal = Rs 3000, Rate = 5%, Time = 2 years.
2. Principal = Rs 5000, Rate = 10 paise per rupee, Time = 2 years.
3. Principal = Rs 625, Rate = 4%, Time = 2 years.
4. Principal = Rs 2000, Rate = 4 paise per rupee, Time = 3 years.
5. Principal = Rs 8000, Rate = 15%, Time = 3 years.
6. Rasheed obtained a loan of Rs 4000 against some security. If the rate of interest is 2.5% per annum compounded annually, find the amount paid by him at the end of 2 years.
7. ✓ Karuna borrowed a sum of Rs 1000 from Varsha for 3 years. If the rate of interest is 10% per annum, compounded annually, find the amount that Karuna will have to pay to Varsha at the end of 3 years.
8. ✓ Abhay lent Rs 8000 to his friend for 3 years at the rate of 5% per annum compound interest. What amount does Abhay get after 3 years?
9. ✓ Find the amount of Rs 2400 after 3 years, when the interest is compounded annually at the rate of 20% per annum.
10. ✓ Helan lent Rs 16000 to Hameed at the rate of $12\frac{1}{2}\%$ per annum compound interest. Find the amount payable by Hameed to Helan after 3 years.

3 Conversion Period

In the examples discussed so far, the interest was calculated on yearly basis, i.e., the interest was compounded annually. However, it is not necessary that the interest be compounded annually. For example, in banks, the interest is, often, compounded twice a year (half yearly or semi-annually), i.e., interest is calculated and added to the principal after every six months. In some financial institutions, interest is compounded quarterly, i.e., four times a year. The period at the end of which the interest is compounded is called the *conversion period*. When the interest is calculated and added to the principal every six months, the conversion period is six months.

Similarly, the conversion period is three months when the interest is calculated and added quarterly. It should be noted that *if no conversion period is specified, the conversion period is taken to be one year.*

In the preceding Section, we obtained a formula to calculate the amount (at compound interest) when the conversion period was 1 year. But how do we calculate the compound interest when the conversion period is not 1 year? The same formula as we obtained earlier, will work in this case also. Only the symbols R and n have a different meaning now. If $R\%$ is the rate of interest per conversion period, and if n denotes the *number of conversion periods*, then the amount A at the end of these n conversion periods is given by

$$A = P (1 + R \times .01)^n$$

Consequently, the compound interest C.I. is given by

$$\text{C.I.} = P \{ (1 + R \times .01)^n - 1 \}$$

As noted earlier $R \times .01$ in both these formula may be replaced by r , where r is the *rate per unit per conversion period*.

Example 1 : Compute the compound interest on Rs 4000 for $1\frac{1}{2}$ years at 10% p.a. compounded half yearly.

Solution : Here, principal $P = \text{Rs } 4000$. Since the interest is compounded half yearly

the number of conversion periods in $1\frac{1}{2}$ years is 3. Also, the rate of interest per

conversion period (6 months) is $(\frac{1}{2} \times 10)\% = 5\%$. Thus, the amount A (in Rs) given by

$$\begin{aligned} A &= P (1 + R \times .01)^n \\ &= 4000 (1 + 5 \times .01)^3 \\ &= 4000 \times (1.05)^3 \\ &= 4630.50 \end{aligned}$$

The compound interest is, therefore, $\text{Rs } (4630.50 - 4000) = \text{Rs } 630.50$.

Example 2 : Compute the compound interest on Rs 2560 for 1 year at $12\frac{1}{2}\%$ p.a. compounded half yearly.

Solution : Here $P = \text{Rs } 2560$, $n = 2$ conversion periods and $R = \frac{1}{2} \times 12.5 = 6.25$ conversion period. Hence, the compound interest

$$\begin{aligned} \text{C.I. (in Rs)} &= P \{ (1 + R \times .01)^n - 1 \} \\ &= 2560 \{ (1 + 6.25 \times .01)^2 - 1 \} \\ &= 2560 \{ (1.0625)^2 - 1 \} \\ &= 2560 \times 2.0625 \times .0625 \\ &= 330 \end{aligned}$$

EXERCISE 7 (c)

1. Find the amount of Rs 1000 in one year at 2% per annum, when the interest is compounded half yearly.
2. ✓ Suraj invested Rs 256 for one year in a fixed deposit scheme. If the rate of interest is $12\frac{1}{2}\%$ per annum compounded half yearly, calculate the amount that he gets after one year.
3. Zaved got a loan of Rs 8000 against his fixed deposits to purchase a scooter. If the rate of interest is 10% per annum compounded half yearly, find the amount that he pays after an year and a half.
4. Find the amount that David would receive if he invests Rs 4096 for 18 months at $12\frac{1}{2}\%$ per annum, the interest being compounded half yearly.
5. ✓ Dalbinder built a hut at a cost of Rs 700. If he borrowed this sum at 20% per annum compounded six monthly, find the compound interest that he pays after one and a half years.
6. ✓ Abha purchased a house from *Avas Parishad* on credit. If the cost of the house is Rs 64000 and the rate of interest is 5% per annum compounded half yearly, find the interest paid by Abha after an year and a half.
7. ✓ Asha lent Rs 40960 to Zubeda to purchase a shop at 12.5% per annum. If the interest is compounded semi-annually, find the interest paid by Zubeda after $1\frac{1}{2}$ years.
8. ✓ Surinder loaned Rs 8192 to Margaret to enable her to purchase a colour TV set. If Surinder charged interest at the rate of 12.5% per annum, compounded half yearly, calculate the amount that Margaret will pay to Surinder after $1\frac{1}{2}$ years..

7.4 To Find the Principal/Time/Rate

The formula

$$A = P(1 + R \times .01)^n$$

connects four quantities A, P, R and n. Similarly,

$$\text{C.I.} = P\{(1 + R \times .01)^n - 1\}$$

connects C.I., P, R and n. Whenever three out of these four quantities are given, the fourth can be found out by simple calculations. In earlier Sections, we calculated A (and C.I.), when P, R and n were given. In this Section, we discuss how to calculate each of the other three quantities when sufficient data is given.

Example 1 : On what sum will the compound interest at 5% per annum for 2 years compounded annually be Rs 164?

Solution : Suppose the principal is Rs 100. Since rate% $R = 5$ and time $n = 2$ years,

$$\begin{aligned} \text{therefore the compound interest (in Rs) on Rs 100} &= P \{(1 + R \times .01)^n - 1\} \\ &= 100 \{(1 + 5 \times .01)^2 - 1\} \\ &= 100 \times 2.05 \times .05 \\ &= 10.25 \end{aligned}$$

Now, if C.I. is Rs 10.25, then $P = \text{Rs } 100$

$$\therefore \text{ if C.I. is Re } 1, \quad \text{then } P = \text{Rs } \frac{100}{10.25}$$

$$\text{and if C.I. is Rs } 164, \quad \text{then } P = \text{Rs } \frac{100 \times 164}{10.25}$$

$$= \text{Rs } 1600$$

Hence, the required sum = Rs 1600.

Example 2 : In what time will Rs 800 amount to Rs 882 at 5% per annum interest compounded annually ?

Solution : Here amount = Rs 882, principal = Rs 800 and rate = 5% per annum. Putting these values in

$$A = P(1 + R \times .01)^n,$$

we get

$$882 = 800 (1 + 5 \times .01)^n$$

$$\text{or, } \frac{882}{800} = (1.05)^n$$

$$\text{Since } \frac{882}{800} = 1.1025 = (1.05)^2,$$

we get $n = 2$. Hence, the required time is 2 years.

Example 3 : Find the rate per cent per annum, if Rs 2000 amount to Rs 2315.25 in an year and a half, interest being compounded six monthly.

Solution : Here $P = \text{Rs } 2000$, $A = \text{Rs } 2315.25$, and $n = 1\frac{1}{2}$ years = 3 half years. Putting these values in

$$A = P(1 + r)^n$$

where r is the rate per rupee per conversion period, i.e., per rupee per half year, we get

$$(1 + r)^3 = \frac{A}{P} = \frac{2315.25}{2000}$$

$$= 1.157625$$

$$= (1.05)^3$$

Therefore, rate = .05 per rupee per six months

= 5 per cent per six months

= 10 per cent per year

EXERCISE 7(d)

[**Note:** In most of the real life problems, lengthy calculations may be involved. To save you from this inconvenience, such computations have been done on a calculator and the approximate values have been given in brackets at the relevant places.]

1. Find the principal, if the compound interest (compounded annually) at the rate of 10% per annum for three years is Rs 331.
2. Rachana borrowed a certain sum at the rate of 15% per annum. If she paid at the end of two years Rs 1290 as interest compounded annually, find the sum she borrowed.
3. What sum will become Rs 4913 in $1\frac{1}{2}$ years, if the rate of interest is $12\frac{1}{2}\%$ compounded half yearly?
4. The interest on a sum of Rs 2000 is being compounded annually at the rate of 4% per annum. Find the period for which the compound interest is Rs 163.20.

5. In what time will Rs 64000 amount to Rs 68921 at 5% per annum, interest being compounded half-yearly ?
6. Mamta invested Rs 1000 in a finance company and received Rs 1331 after 3 years. Find the rate of interest per cent per annum compounded annually.
7. Ramdeen purchased *Rahat Patras* for Rs 1000, a sum which will fetch him Rs 2000 after 5 years. Find the rate of interest if the interest is compounded half-yearly. [Given that $^{10}\sqrt{2} = 1.072$]
8. Preeti purchased one six years National Savings Certificate for Rs 1000. After six years she got Rs 2015. Find the rate of interest, if the interest is compounded half-yearly. [Given that $(2.015)^{\frac{1}{2}} = 1.06012$.]
9. Find the compound interest at the rate of 5% for three years on that principal which in three years at the rate of 5% per annum gives Rs 1200 as simple interest.
10. Kamala borrowed from Ratan a certain sum at a certain rate for two years simple interest. She lent this sum at the same rate to Hari for two years compound interest. At the end of two years she received Rs 210 as compound interest, but paid Rs 200 only as simple interest. Find the sum and the rate of interest.
[Hint : If r is the rate per rupee per annum, then $P r = 100$, and $P\{(1 + r)^2 - 1\} = 210$]

7.5 Growth and Depreciation

We shall now illustrate how formulae like the one used for calculating amount and compound interest are found useful in some other daily life situations. In particular, we shall see how the growth of population and the depreciation or decrease in the value of a property can be calculated by using such a formula. We illustrate these and other uses with the help of the following examples :

Example 1 : The population of a town is increasing at the rate of 5% per annum. What will be the population of the town on this basis after two years, if the present population is 16000 ?

Solution : Since the population is increasing at the rate of 5% per annum, the increase in population after one year will be $16000 \times 5 \times .01$. Hence, the population at the

end of the first year will be $16000 + 16000 \times 5 \times .01 = 16000(1 + .05) = 16000 \times 1.05$. During the second year, the increase in population will be 5% of the population at the end of the first year. Thus, this increase is $16000 \times 1.05 \times .05$. Hence, the population at the end of the second year will be

$$\begin{aligned} 16000 \times 1.05 + 16000 \times 1.05 \times .05 &= 16000 \times 1.05 \times (1 + .05) \\ &= 16000 \times 1.05 \times 1.05 = 17640 \end{aligned}$$

We can calculate the population directly by using the formula

$$A = P(1 + R \times .01)^n,$$

where P is the present population, $R\%$ is the rate of increase per annum and A is the population after n years. Here $P = 16000$, $R = 5$ and $n = 2$. Therefore, the population after 2 years will be

$$16000(1 + 5 \times .01)^2 = 16000 \times (1.05)^2 = 17640$$

Example 2 : The cost of a refrigerator is Rs 7812.50. If its value depreciates at the rate of 12% per annum, what will be the total depreciation in its value at the end of 3 years?

Solution : Here the annual decrease in value is 12%, i.e., if the value of the refrigerator were Rs 100 now, after one year it will be Rs $(100 - 12) =$ Rs 88 only. Therefore, the value which is Rs 7812.50 now, will be Rs $7812.50 \times 88 \times .01$ after one year. After two years, the value will reduce to $88 \times .01$ times of the value at the end of the first year, i.e., it will be Rs $7812.50 \times 88 \times .01 \times 88 \times .01 =$ Rs $7812.50 \times (88 \times .01)^2$. Finally at the end of the third year, the value will reduce to Rs $7812.50 \times (88 \times .01)^3 =$ Rs 5324. Hence, the total depreciation = Rs $(7812.50 - 5324) =$ Rs 2488.50.

Here also we can use a formula like the one for compound interest. Here P , the present value is Rs 7812.50 and the rate (per cent per annum) is -12 . Here we put a minus sign before 12, because the value is *decreasing*. Hence, the total depreciation (like compound interest) is given by

$$\begin{aligned} &P\{(1 + R \times .01)^n - 1\} \\ &= 7812.50 \{(1 - 12 \times .01)^3 - 1\} \\ &= 7812.50 \times \{(.88)^3 - 1\} \\ &= 7812.50 \times (-0.318528) \\ &= -2488.50 \end{aligned}$$

Negative sign here indicate the decrease in value. Hence, the total depreciation = Rs 2488.50.

Example 3 : 6400 workers were employed to construct a river bridge in four years. At the end of the first year, 25% workers were retrenched. At the end of the second year, 25% of those working at that time were retrenched. However, to complete the project in time, the number of workers was increased by 25% at the end of the third

year. How many workers were working during the fourth year ?

Solution : During the first two years, the number decreased at the rate of 25% per annum. Hence, the number of workers at the end of the second year was $6400(1 - 25 \times .01)^2$. At the end of the third year, the number was increased by 25%. Therefore, the number of workers working during the fourth year is

$$6400(1 - 25 \times .01)^2(1 + 25 \times .01) \\ = 6400 \times .75 \times .75 \times 1.25 = 4500$$

Example 4 : Ashish opened a bookshop with an initial investment of Rs 32000. In the first year, he incurred a loss of 5%. However during the second year, he earned a profit of 10% which in the third year rose to $12\frac{1}{2}\%$. Calculate his net profit for the entire period of three years.

Solution : His assets at the end of three years are worth

$$\text{Rs } 32000(1 - 5 \times .01)(1 + 10 \times .01)(1 + 12.5 \times .01) \\ = \text{Rs } 32000 \times .95 \times 1.1 \times 1.125 \\ = \text{Rs } 37620$$

$$\text{Hence, net profit} = \text{Rs } (37620 - 32000) \\ = \text{Rs } 5620$$

Example 5 : In 1981, the population of India was 6.7×10^8 . After $2\frac{1}{2}$ years, the population rose to 7.128×10^8 . Find the half yearly rate of growth of the population. [Given that $(7.128 \div 6.7)^{\frac{1}{5}} = 1.01246$.]

Solution : Let $R\%$ be the rate of growth per half year. Then

$$7.128 \times 10^8 = 6.7 \times 10^8 (1 + R \times .01)^5$$

$$\text{or,} \quad (1 + R \times .01) = \left(\frac{7.128}{6.7} \right)^{\frac{1}{5}} \\ = 1.01246 \quad (\text{given})$$

$$\text{or,} \quad R = 1.246$$

Hence, rate of growth = 1.246% per half year.

EXERCISE 7(e)

1. The value of a residential flat constructed at a cost of Rs 100000 is — depreciating at the rate of 10% per annum. What will be its value 3 years after construction ?

2. Ms Cherian purchased a boat for Rs 16000. If the cost of the boat is depreciating at the rate of 5% per annum, calculate its value after 2 years.
3. The *Nagarpalika* of a certain city started a campaign to kill stray dogs which numbered 1250 in the city. As a result the population of stray dogs started decreasing at the rate of 20% per month. Calculate the number of stray dogs in the city three months after the campaign started.
4. The population of a certain city is 125000. If the annual birth rate is 3.3% and the annual death rate is 1.3%, calculate the population after 3 years.
5. 8000 blood donors were registered with a charitable hospital. Some student organizations started mobilizing people for this noble cause. As a result, the number of donors registered increase at the rate of 10% per half year. Find the total number of new registrants during $1\frac{1}{2}$ years.
6. Shakeel set up a factory by investing Rs 40000. During the first three successive years, his profits were 5%, 10% and 15% respectively. If each year the profit was on previous year's capital, calculate his total profit.
7. The cost of a certain type of a TV set of a certain company was quoted as Rs 7500 at the beginning of 1986. After a few months, the price was hiked by 5%. Because of decrease in demand, the cost was reduced by 4% in 1987. However, because of some taxes in 1988 budget, the price was again hiked by 5%. What is the cost of the TV set at the end of 1988?
8. In the month of January, the Railway Police caught 4000 ticketless travellers. In February, the number rose by 5%. However, due to constant vigil by the Police and the Railway staff, the number reduced by 5% in March and in April it further reduced by 10%. What was the total number of ticketless travellers caught in the month of April?
9. The population of Pakistan in 1980 was 7.95×10^7 . After 3 years, the population became 8.65×10^7 . Find the annual rate of increase, if it

is given that $\left(\frac{8.65}{7.95}\right)^{\frac{1}{3}} = 1.02853$.

$12\frac{x}{6}$



10. The production of iodised salt in India in 1985 was 288000 tonnes. If the production in 1988 is 1700000 tonnes, what was the annual rate

of increase in production. [Given $\left(\frac{1700}{288}\right)^{\frac{1}{3}} = 1.8072524$]

11. The wholesale price index at the beginning of September 1987 was 400. After 10 months at the end of June 1988, the index rose to 435. What is the monthly rate of increase in the price index ?

[Given $\left(\sqrt[10]{\frac{435}{400}}\right) = 1.00842.$]

7.6 Public Sector Undertakings

An industry, big or small, may be established by one person or a group of persons. It can be established with or without the help of the Government. If an industry is owned and managed by people without any major financial or administrative control of the Government, then it is called a *private industry*, or a *private enterprise* or an industry in the *private sector*.

There are, however, some undertakings over which the Government have complete financial as well as administrative control. They are called *public sector undertakings*. Some key public sector undertakings in the country are Hindustan Aeronautics Ltd, Life Insurance Corporation of India, National Textile Corporation of India, Bharat Heavy Electricals Ltd, Hindustan Machine Tools, Oil and Natural Gas Commission, Steel Authority of India and National Thermal Power Corporation. All these are Central Government undertakings, i.e., these public sector undertakings are run under the control of the Government of India. Besides these, there are many state government undertakings as well, like Gujarat Communications and Electronics Ltd (Gujarat Government undertaking), Tamil Nadu Minerals Ltd (Tamil Nadu Government undertaking), UP Jal Nigam (UP Government undertaking), etc.

Public sector has been playing a vital role in the economic development of the country. Through more than 200 enterprises that the Indian government owns, it has helped India achieve self-sufficiency in many vital fields. Through investment of more than five thousand crores in these industries, the Government has generated more than 50 lakh jobs. During 1986-87, the total turnover of these enterprises was to the tune of Rs 69000 crores. It fetched a net profit of about Rs 1700 crores. These

figures show how important these enterprises are for our country. In fact in a developing country like India, public enterprises form backbone of the economy.

As public sector enterprises are Government undertakings, finances for these industries come mainly from the Government exchequer. Some other sources of funding these enterprises are loans from international financial institutions, loans from Indian banks and borrowing from general public. Borrowing from the public is done through the issue of various types of bonds. These bonds carry a specific rate of interest.

Example 1 : A public sector undertaking secured a 20-year loan of Rs 25 crores from a foreign bank through the Government of India. If the loan carried interest at the rate of 12.5% per annum, interest compounded annually, find the total compound interest payable by the undertaking. [Given that $(1.125)^{20} = 10.5451$.]

Solution : Here $P = \text{Rs } 25 \text{ crores}$, $R = 12.5$ and $n = 20$. Hence, the compound interest is given by

$$\begin{aligned} \text{C.I.} &= \text{Rs } 25 \{ (1 + 12.5 \times .01)^{20} - 1 \} \text{ crores} \\ &= \text{Rs } 25 \{ (1.125)^{20} - 1 \} \text{ crores} \\ &= \text{Rs } 25 \times 9.5451 \text{ crores} \\ &= \text{Rs } 238.6275 \text{ crores} \end{aligned}$$

Example 2 : National Textile Corporation (NTC) of India imported 200 looms from USSR of value Rs 5.21 crores on ten - year credit at 4% rate of interest per annum. Find the compound interest paid by NTC at the end of 10 years, if the interest is compounded annually. [Given that $(1.04)^{10} = 1.48024$.]

Solution : Here the compound interest is given by

$$\begin{aligned} \text{C.I.} &= \text{Rs } 5.21 \{ (1 + 4 \times .01)^{10} - 1 \} \text{ crores} \\ &= \text{Rs } 5.21 \times .48024 \text{ crores} \\ &= \text{Rs } 2.5020504 \text{ crores} \end{aligned}$$

Thus, the total compound interest paid by NTC is Rs 2 crore 50 lakh 20 thousand five hundred and four only.

EXERCISE 7(f)

1. Himachal Pradesh State Electricity Board issued in July 1988, 20 - year bonds worth Rs 6.25 crores. The issue price of each bond is Rs 100 and it carries an annual interest of 11.5%, compounded half-yearly. Veerbhadra invested Rs 5000 in these bonds. Find the amount

that he gets on maturity of the bonds in 2008. [Given that $(1.0575)^{40} = 9.35869$.]

2. Gomang invested Rs 1000 in State Government 10 - year loan scheme, where the interest is compounded half yearly at the rate of 11.5% per annum. Find the interest that Gomang gets after 10 years. [Given that $(1.0575)^{20} = 3.0592$.]
3. The Industrial Development Bank of India (IDBI) issued in October 1988 its 50th series of bonds for Rs 3187.20 crores. The bonds with maturity period of 20 years carry interest at the rate of 11.5% per annum, compounded half-yearly. Uma Shankar invested Rs 10000 in these IDBI bonds. Find the amount that he gets on maturity. [Given that $(1.0575)^{40} = 9.35869$.]
4. In 1980-81, Oil and Natural Gas Commission (ONGC) earned net profit of Rs 46.57 crores. After 7 years, in 1987-88, the profit rose to Rs 1508 crores. Find the rate (per cent per annum) of growth of profit, compounded annually. [Given that $\left(\frac{1508}{46.57}\right)^{\frac{1}{7}} = 1.64345$.]

7.7 Miscellaneous Problems

Example 1 : Given that carbon 14 decays at a constant rate in such a way that it reduces to 50% in 5568 years, find the age of an old wooden piece in which the carbon is only $12\frac{1}{2}\%$ of the original.

Solution : Suppose r denotes the rate of decay per unit per year. Then, according to the data given,

$$(1 - r)^{5568} = \frac{50}{100} = \frac{1}{2}$$

Let the age of the wooden piece be n years. Then

$$\begin{aligned} (1 - r)^n &= \frac{25}{2 \times 100} = \frac{1}{8} = \left(\frac{1}{2}\right)^3 = [(1 - r)^{5568}]^3 \\ &= (1 - r)^{16704} \end{aligned}$$

Hence, $n = 16704$. Thus, the age of the wooden piece is 16704 years.

Remark: This method has been used to determine the age of the Earth.

Example 2 : The world's population in 1982 was about 450 crores. Given that the population grows at a constant rate and doubles itself in 38 years, what will be the population of the world in 2020 A.D. ? The total surface (land) area of the Earth is about 150 million sq. km. If the population continues to grow at the same rate, what is approximate land that would be available per person in 2362 A.D.?

Solution : Let the annual growth rate of the population be $R\%$. It is given that

$$(1 + R \times .01)^{38} = 2$$

Hence, the population in 2020 A.D., i.e., 38 years after 1982 will be

$$\begin{aligned} &4.5 (1 + R \times .01)^{38} \text{ billions} \\ &= 4.5 \times 2 \text{ billions} \\ &= 9.0 \text{ billions} = 9.0 \times 10^9 \end{aligned}$$

In 2362 A.D., i.e., 380 years after 1982, the population will become

$$\begin{aligned} &4.5 (1 + R \times .01)^{380} \text{ billions} \\ &= 4.5 \{(1 + R \times .01)^{38}\}^{10} \text{ billions} \\ &= 4.5 \times 2^{10} \text{ billions} \\ &= 4.5 \times 2^{10} \times 10^9 \end{aligned}$$

Therefore, the land per person will be

$$\begin{aligned} &= \frac{150 \times 10^6}{4.5 \times 2^{10} \times 10^9} \text{ km}^2 \\ &= \frac{150 \times 10^6 \times 10^6}{4.5 \times 2^{10} \times 10^9} \text{ m}^2 \\ &= 1.085 \text{ sq. m} \end{aligned}$$

Example 3 : The money invested in *Indira Vikas Patras* doubles itself in five years. What is the rate of interest per annum if (a) the interest is simple interest (b) the interest is compounded annually (c) the interest is compounded half-yearly. [Given that $\sqrt[5]{2} = 1.1487$ and $\sqrt[10]{2} = 1.07177$.]

Solution : (a) Suppose the principal is Rs 100. Then the amount after 5 years is Rs 200. If $R\%$ is the rate of simple interest per annum, then

$$\frac{100 \times R \times 5}{100} = 100$$

$$\text{or, } R = 20$$

(b) Let $R'\%$ is the rate of interest per annum, when the interest is compounded

annually. Then

$$200 = 100 (1 + R' \times .01)^5$$

$$\begin{aligned} \text{or, } R' &= (\sqrt[5]{2} - 1) \times 100 \\ &= (1.1487 - 1) \times 100 \\ &= 14.87 \end{aligned}$$

(c) If we denote by $R''\%$, the rate when interest is compounded half-yearly, then

$$200 = 100 \left(1 + \frac{1}{2} \times R'' \times .01\right)^{10}$$

$$\begin{aligned} \text{or, } R'' &= 2 \times (\sqrt[10]{2} - 1) \times 100 \\ &= 2 \times (.07177) \times 100 \\ &= 14.354 \end{aligned}$$

Hence, the respective rates of interest are (a) 20%, (b) 14.87% and (c) 14.354%

EXERCISE 7 (g)

1. Foodgrain production in India in 1984 was 1520 lakh tonnes. If the production is increasing at a constant rate of 2.8% per annum, what the production is expected to be in 2000 A.D. on that basis? [Given that $(1.028)^{16} = 1.55557$.]
2. According to a UN survey on population, there was only one city in 1950 whose population was above 40 lakhs. In 2000 A.D., this number is expected to be 135. Find the annual rate at which the number of such cities is increasing. Assume that the rate of growth is constant. [Given that $(135)^{.02} = 1.10308$.]
3. In 1961, the total investment (in Rs) in child welfare scheme through Integrated Child Development Schemes (ICDS) was 3 crores. In 1986, the investment rose to 3000 crores. Find the constant annual rate at which the investment in ICDS increased. [Given that $(1000)^{.04} = 1.31826$.]
4. In 1965, India exported 60 lakh books. In 1985, the figure rose to 25 crores. Find the constant annual rate at which India's book export increased. [Given that $\left(\frac{250}{6}\right)^{.05} = 1.205$.]

5. The half life of Uranium 233 is 160000 years, i.e., Uranium 233 decays at a constant rate in such a way that it reduces to 50% in 160000 years.
 - In how many years will it reduce to 25% ? The half life of Plutonium 242 is 380000 years. In how many years will it be reduced by 75% ?
6. In 1963, fertiliser consumption in India was 6 kg per hectare. In 1988, it reached the level of 48 kg per hectare. Find the constant annual rate at which fertiliser consumption per hectare increased during this period. [Given that $(8)^{.04} = 1.08673$.]
7. The population of Andhra Pradesh was 5.4×10^7 in 1980. If the population is growing at a constant rate of 2.4% per annum, find the population in 2000 A.D.. [Given that $(1.024)^{20} = 1.60694$.]

Things to Remember

1. The period after which the interest is added to the principal is called conversion period.
2. $A = P (1 + R \times .01)^n$, where $R\%$ is the rate per conversion period and n is the number of conversion periods.
3. $C.I. = A - P$
 $= P\{(1 + R \times .01)^n - 1\}$
4. In the case of depreciation or decay, the rate R is replaced by $-R$ in the formulae like $A = P (1 + R \times .01)^n$.

CHAPTER EIGHT

General Banking Facilities

Introduction

HAVE YOU SEEN a bank (in fact, a branch of a bank)? Most of you must have! There might be one on your way to school. What are these banks? What is carried out there?

Some people have some spare money. Instead of keeping it with them and taking the risk of losing it by theft etc., they generally like to deposit it with a bank and earn interest on it. On the other hand, businessmen may borrow money from the banks for their business needs. Thus, in a way banks are deposit taking and money lending institutions. However, banks also perform various other functions and almost every section of the society deals with banks in one way or the other.

Some salaried people receive their pay through banks. In big cities, school fees, electricity and water bills, rent and Government loan instalments, etc. are paid by depositing the money in banks. Besides, banks also perform certain other functions for the benefit of the society. For example, they give loans at concessional rates to small farmers, petty shopkeepers, qualified technical personnel, educated unemployed persons to start business or industry; to the economically weaker sections of society to let them improve their lot and to the handicapped persons and widows to help them earn their livelihood.

Apart from the above, some other functions of the banks are the collection of bills of exchange, promissory notes and hundies, transfer of credit, issue of traveller's cheques, etc.

8.1 Opening a Bank Account

As mentioned earlier, one of the main functions of a bank is to get deposits from the public. When we deposit money in a bank for the first time, we have to open an account with the bank. For this purpose, we apply to the bank on a prescribed

Application form available from the bank (Fig. 8.1). We put our specimen signatures on this form. These signatures are very important. Each time we go to the bank to

Account Opening Form For Individuals (Current/Savings Bank/Recurring Deposit Accounts)		STATE BANK OF INDIA N.C.E.R.T., New Delhi		ACCOUNT NUMBER	
Please open the account ticked () below :				RECURRING DEPOSIT ACCOUNT	
☐ CURRENT ACCOUNT	☐ SAVINGS BANK ACCOUNT (WITH CHEQUE FACILITY)	☐ SAVINGS BANK ACCOUNT (WITHOUT CHEQUE FACILITY)	☐ PERIOD MTHS.	Rs.	
Full Name (Block Letters)		Nationality	Occupation	Address	
A					
B					
C					
Date of Birth In case of Minor	Mode of Operation/Repayment* (In case of Joint Accounts)	Name and Address of Introducer			
I/We agree to abide by the Bank's rules relating to Current A/Cs/Savings Bank A/Cs/R.D. A/Cs.				FOR OFFICE USE (Signatures of verifying official)	
A					
B					
C					
Date				19.....	
I know the above applicant(s) personally.		Open	Account Opened	Numbers of Cheque Book Issued	
Signature of Introducer	A/c No. (if any)	Br. Manager	Ledger Keeper	Officer	From To
Please Indicate a suitable choice viz Either or Survivor, Former/Latter or Survivor etc.					
*R.F. 38					

Fig. 8.1

Withdraw money from our account with the bank, the bank officials tally our signature with the specimen signature in their record. In case, the signature does not tally, the bank may well refuse to make payment.

A bank may operate different types of Accounts. Some such Accounts are:

- Savings Bank Account
- Current Account
- Fixed Deposit Account
- Recurring(Cumulative) Deposit Account

Savings bank account is basically meant for people like salaried people, people belonging to small and middle income groups, etc. The main purpose of such an account is to encourage the habit of savings. This account may be opened with a minimum deposit of Rs 20 (State Bank). For the money that you keep in this account, bank pays some interest. The current rate of interest is 5% per annum.

Current Account is usually maintained by businessmen, companies, corporations, etc. who instead of handling cash, operate through this account. There are some facilities which are available in current account but not in savings account. For example, there is no restriction on the amount or on the number of withdrawals from this account. However, no interest is paid for the money kept in this account. In fact, in certain cases, bank charges incidental expenses from the account-holder.

Fixed Deposit Account is an account in which money is deposited for a fixed period. Money can be withdrawn from this account normally after the fixed period which is specified at the time of opening the account. This period may be chosen by the depositor to suit his/her needs. It may vary from few days to several years. In the case of fixed deposit accounts, the bank is sure as to how long the money is going to remain with the bank. Therefore, the bank can utilize this money more freely than the money kept in savings account. Consequently, banks offer higher rate of interest on such deposits. The interest rate depends upon the period for which deposits have been made.

The rates* of interest on fixed deposits w.e.f. 28-3-1989 are as follows:

(a) For 46 days to 90 days	6%
(b) For 91 days and above but less than 1 year	8%
(c) For 1 year and above but less than 2 years	9%
(d) For 2 years and above	10%

Recurring (Cumulative) Deposit Account is somewhat like the fixed deposit account. A depositor opening this account chooses a specified amount (normally multiple of Rs 5) and deposits that amount in this account every month for a period chosen by him at the time of opening the account. This period may range from a few months to 10 years. The rate of interest on this account is nearly equal to the rate on fixed deposits. Usually bank publishes tables which show the amount of instalment, the number of months for which the depositor contributes and the amount that the depositor would get after the expiry of the fixed period. Such tables are revised from time to time depending on the changes in the rate of interest. One such table in use in 1988 is given on page 169.

Whenever an Account such as Savings Bank Account, Current Account is opened, the bank issues to the customer a notebook called *passbook*. The passbook contains the details of deposits and withdrawals recorded datewise. Customers are required to present the passbook to the bank for updating the entries regarding deposits and withdrawals as also those that relate to interest added by the bank in the account.

Certain banking facilities are provided through post offices also. In post office savings bank, the present rate of interest is $5\frac{1}{2}\%$ per annum.

Minors' Account

A person who is below 18 years of age is said to be a minor. A minor is eligible to open a Savings bank Account with a bank. He, however, cannot open a Current Account.

* These rates of interest keep on changing.

issued in different denominations, e.g., Rs 50, Rs 100 or Rs 500. A person can buy any number of travellers cheques by depositing equivalent amount with the bank. At the time of purchasing these cheques, the buyer puts his signatures on the prescribed

Specimen of a Travellers' Cheque

STATE BANK OF INDIA
TRAVELLERS' CHEQUE

X D 310197

WHEN COUNTERSIGNED BELOW
WITH THIS SIGNATURE

.....19.....

BEFORE CASHING WRITE
HERE PLACE AND DATE

STATE BANK OF INDIA
STRAND ROAD, CALCUTTA (India)

PAY THIS CHEQUE
TO THE ORDER OF.....

IN INDIA	IN OTHER COUNTRIES
RUPEES FIFTY	Negotiable at the Current Buying Rate for Bankers' Cheques on India
COUNTERSIGN HERE IN PRESENCE OF PERSON CASHING	 Managing Director

Fig. 8.3

place on the cheques. At the time of encashment he is required to sign again on the cheques. These signatures must tally with his earlier signatures on the cheques. Travellers' cheques are encashable at any branch of the issuing bank. Many hotels, shopping centres, stores etc., also accept travellers' cheques from their customers. For this purpose, they display sign board of the following type:

We Accept Travellers' Cheques

Safe Custody of Valuables (Lockers)

Though banks deal with money, other valuable items like jewellery, important documents etc., can also be kept in safe custody of a bank. This facility is being provided by many leading banks at their selected branches. By paying the prescribed rent, one may hire a locker, a box in an almirah kept in *strong room* of the bank. This locker can be opened by the application of double keys. One key is retained by the person who hires a locker, and the other key, the masters key, is retained in the bank. No bank official knows the contents of the lockers. Thus, articles kept in a locker remain confidential even from bank officials.

8.2 Operating a Bank Account

Operating a bank account means depositing money and withdrawing money from the account according to one's need and convenience. To make a deposit in the bank, a customer uses a pay-in-slip [Figs. 8.4 (a) and (b)], a form available from the bank free of charge. The slip consists of two parts, a foil and a counterfoil separated by a perforated line. The two foils contain blank space for providing information like the

रोकड़ Cash भारतीय स्टेट बैंक
STATE BANK OF INDIA
N.C.E.R.T., New Delhi

भारतीय स्टेट बैंक को
के बचत खाता नं.
में जमा के लिए रुपये
रिफ्ट गर।
Paid to the State Bank of India for credit of
Savings Bank Account No.
of
Rupees
रु./Rs.
रोकड़िया सारणी क्र.
Cashier Scroll No.
रोकड़िया
Cashier
प्रधान रोकड़िया
Head Cashier

सी.ओ.एस. 36
बचत खाता जमा पर्ची
SAVINGS BANK PAY-IN-SLIP

	रु./Rs.	पे./P.
100x		
50x		
20x		
10x		
5x		
2x		
1x		
रुपये/Rupees		
सिक्के/Coins		
कुल योग/Total		

रोकड़िया
Cashier
प्रधान रोकड़िया
Head Cashier

भारतीय स्टेट बैंक
STATE BANK OF INDIA
N.C.E.R.T., New Delhi

खाता नं.
Account No.
खाता बही पृष्ठ
Ledger Foilo
बचत खाते में रुपये
के
जमा किए गए।
Paid into the CREDIT of the Savings Bank
Account of
Rupees
रोकड़िया सारणी क्र.
Cashier Scroll No.
जोती बही क्र.
Jotting Book No.
जमाकर्ता
Depositor (s)

Fig. 8.4(a)

date of deposit, the amount of deposit, details and particulars of deposit (cash or cheque), the name of account-holder and the number of account in which the amount is to be deposited, etc. The foil remains with the bank whereas the counterfoil is returned by the bank to the depositor, as an acknowledgement of the receipt of the deposit. The amount thus deposited is credited in (i.e. put into) the account.

ट्रांसफर
TRANSPER _____ 19 _____
खाता क्र. Account No. _____
खाता बही पृष्ठ Ledger Folio _____
भारतीय स्टेट बैंक को पीछे लिखे विवरण
के अनुसार भुगतान करने पर,
Paid to the State Bank of India
for credit of the Savings Bank
Account of _____ के नाम
जमा के लिये
रुपये Rupees _____
दिये गए।
रु. Rs. _____
पासकर्ता अधिकारी
Passing Official _____

भारतीय स्टेट बैंक STATE BANK OF INDIA
N.C.E.R.T., New Delhi
बचत बैंक खाता जमा पर्ची
SAVINGS BANK PAY-IN-SLIP
केवल चेक, ड्राफ्ट आदि के लिए
For Cheques, Drafts, etc. only
खाता क्र.
Account No. _____
खाता बही पृष्ठ Ledger Folio _____ 19 _____
पीछे लिखे विवरणों के अनुसार भुगतान करने पर
Paid into the Credit of the Savings Bank Account of _____ के बचत बैंक खाते में
रुपये Rupees _____
_____ जमा किए गये।
रु. Rs. _____ on realisation as per particulars overleaf
पासकर्ता अधिकारी Passing Official _____ जमाकर्ता BY _____
अन्तरण सारणी Scroll Transfer _____

Fig. 8.4(b)

To withdraw money, two modes are used— *withdrawal forms* and *cheques*. Withdrawal form (Fig. 8.5) is a slip available from the bank. The customer fills in it

संक्षिप्त हस्ताक्षर
Initials
1-89/5,000 सी.ओ.एस./C.O.S. 161ए
खाता बही पृष्ठ
Ledger Folio

सावधानी : यह बचत बैंक निकासी आदेश फार्म चेक नहीं है। इस फार्म के साथ पास-बुक रहना अनिवार्य है अन्यथा भुगतान प्राप्त नहीं होगा।
CARE : This Savings Bank Withdrawal Order Form is Not a Cheque. Unless this form is accompanied with Pass Book payment will be refused.
प्रश्नी/To: भारतीय स्टेट बैंक STATE BANK OF INDIA
N.C.E.R.T., New Delhi
बचत बैंक/SAVINGS BANK
Date _____
कृपया स्वयं या धारक को Please pay self or bearer
रुपये _____ देकर
Rupees
तथा राशि को मेरे/हमारे बचत बैंक खाता क्र. _____ में नामें हलिये
and debit the amount to my/our Savings Bank Account No. _____
नकद भुगतान
जमा प्रमाणक जमाकर्ता/Depositor _____

Fig. 8.5

appropriate details and presents it to the bank officials for payment. Normally passbook is also required along with withdrawal forms.

A cheque (Fig. 8.6) like withdrawal form is also a printed proforma issued by the bank. However, cheques are numbered and are issued to the customers in the form of a cheque book containing 10, 20, 25 or 100 cheques. A cheque is an unconditional

order to the bank and is always payable on demand. Unless the word 'bearer' printed on the cheque is crossed out, the cheque is called a *bearer cheque*. A bearer cheque can be encashed by any one irrespective of the name written on the cheque (in front

19

PAY

या धारक को OR BEARER

रुपये RUPEES

अदा करें

₹.Rs.

खा.सं.
A/c No.

ब.प.
LF.

ह.ह.
INTLS.

भारतीय स्टेट बैंक STATE BANK OF INDIA

MSB.65 एन.सी.ई.आर.टी., नई दिल्ली NCERT, New Delhi

⑈ 775686⑈ 100002078⑈ 10

Fig. 8.6

of pay.....). The bank cannot deny the payment of a bearer cheque even if it has fallen in wrong hands. To avoid this risk, a cheque may be made an *order cheque*. This can be done by just crossing out the word 'bearer' on the cheque form. The payment of an order cheque is made only to that person in whose name the cheque is issued. The risk involved in issuing order cheque is considerably less, yet it is not completely risk free. To minimize the risk further a cheque may be *crossed*. The payment of a crossed cheque is not made in cash. Instead, the amount is deposited in the account of the person in whose name the cheque is issued. To cross a cheque, we merely draw two parallel slant lines on the face of the cheque. It is customary to write '& co' between the lines. Sometimes 'Not Negotiable', 'A/c Payee only', etc. are written between the lines (Fig. 8.7). These are extra precautions to prevent any fraudulent misuse of the cheque.

27 FEB 1989 19

₹.Rs. 900.00

को या उनके आदेशपर OR ORDER

अदा करें

खा.सं.
A/c No.

ब.प.
LF.

ह.ह.
INTLS.

भारतीय स्टेट बैंक STATE BANK OF INDIA

MCAO/75 एन.सी.ई.आर.टी., नई दिल्ली N. C. E. R. T., NEW DELHI

⑈ 22264⑈ 100002078⑈ 11

Fig. 8.7

Sometimes, in the date column of a cheque we write a date different from the one when cheque is written. For example, on a cheque written on 01.11.88, we may write 11.09.88 or 27.01.89. In the first case, it is called an *antedated* cheque, and in the second case, we call it a *postdated* cheque. Writing such cheques is quite common among traders and businessmen.

Suppose an order cheque is drawn in the name of Amar, who for some reasons wants Akbar to receive its payment from the bank. Amar then writes on the backside of the cheque 'Pay to Akbar' and puts his signature below it. In this case, we say that the cheque has been *endorsed* in the name of Akbar. After endorsement Akbar becomes entitled to receive the payment after presenting it to the bank. However, Akbar, if so desires, may endorse it in the name of Anthony. Thus, a cheque may be endorsed several times. The one whose name appears in the end is entitled for the payment.

8.3 Use of Tables

In banks (and in most other financial institutions) interest is calculated with the help of previously prepared tables. One such table (Table A) is given on page 169. Using this table, we can find the maturity value of various recurring deposits. For fixed deposits also we use similar tables. The Table B on page 170 shows the growth of principal when the interest at the rate of 10% per annum is compounded half-yearly. We can use it to calculate the amount of any principal for any number of years (upto 20). The Table C on page 171 shows the growth of principal if the interest is compounded quarterly. We explain the use of these tables by some examples.

Example 1 : Abha saves Rs 25 every month from her monthly pocket allowance. If she puts this saving in a recurring deposit account for 7 years, what is the amount that she gets on maturity ?

Solution : See Table A. The amount payable to Abha is given in the row headed by 25 (her monthly instalment), and the column headed by 84 ($= 7 \times 12$, the number of months for which she made the deposit). The figure is 3039.20. Hence, on maturity she gets Rs 3039.20.

Example 2 : Ujjawala needs Rs 3700 after 3 years to purchase a washing machine. What is the minimum monthly instalment for a recurring deposit that will give her approximately this amount after 3 years ?

Solution : We examine the column headed by 36 ($= 3 \times 12$) in the Table A. The amount which is nearest (and more than) Rs 3700 is Rs 3786.66 and it appears in the row headed by 90. Hence, the minimum monthly instalment is Rs 90.

Table A

With effect from 1-4-1987
Total Amount Repayable at Maturity

RECURRING DEPOSIT

Monthly Instalment	6 months	9 months	12 months	24 months	36 months	48 months	60 months	72 months	84 months	96 months	108 months	120 months
20	122.80	186.08	251.92	532.88	841.48	1,182.16	1,558.16	1,973.20	2,431.36	2,937.08	3,495.24	4,111.40
25	153.50	232.60	314.90	666.10	1,051.85	1,477.70	1,947.70	2,466.50	3,039.20	3,671.35	4,369.05	5,139.25
30	184.20	279.12	377.88	799.32	1,262.22	1,773.24	2,337.24	2,969.80	3,647.04	4,405.62	5,242.86	6,167.10
35	214.90	325.64	440.86	932.54	1,472.59	2,068.78	2,726.78	3,453.10	4,254.88	5,139.89	6,116.67	7,194.95
40	245.60	372.16	503.84	1,065.76	1,682.96	2,364.32	3,116.32	3,946.40	4,862.72	5,874.16	6,990.48	8,222.80
45	276.30	418.68	566.82	1,198.98	1,893.33	2,659.86	3,505.86	4,439.70	5,470.56	6,608.43	7,864.29	9,250.65
50	307.00	465.20	629.80	1,332.20	2,103.70	2,955.40	3,895.40	4,933.00	6,078.40	7,342.70	8,738.10	10,278.50
60	368.40	558.24	755.76	1,598.64	2,524.44	3,546.48	4,674.48	5,919.60	7,294.08	8,811.24	10,485.72	12,334.20
70	429.80	651.28	881.72	1,865.08	2,945.18	4,137.56	5,453.56	6,906.20	8,509.76	10,279.78	12,233.34	14,389.90
80	491.20	744.32	1,007.68	2,131.52	3,365.92	4,728.64	6,232.64	7,892.80	9,725.44	11,748.32	13,980.96	16,445.60
90	552.60	837.36	1,133.64	2,397.96	3,786.66	5,319.72	7,011.72	8,879.40	10,941.12	13,216.86	15,728.58	18,501.30
100	614.00	930.40	1,259.60	2,644.40	4,207.40	5,910.80	7,790.80	9,866.00	12,156.80	14,685.40	17,476.20	20,557.00

10% p.a.

Table B
COMPOUND INTEREST TABLES
 Showing growth of principal compounded HALF YEARLY

10% p.a.

1 Half year	2 Half years	3 Half years	4 Half years	5 Half years	If Principal is	6 Half years	7 Half years	8 Half years	9 Half years	10 Half years
Rupees	Rupees	Rupees	Rupees	Rupees		Rupees	Rupees	Rupees	Rupees	Rupees
1.050000	1.102500	1.157625	1.215506	1.276282	Rs. 1	1.340096	1.407100	1.477455	1.551328	1.628895
2.100000	2.205000	2.315250	2.431012	2.552563	2	2.680191	2.814201	2.954911	3.102656	3.257789
3.150000	3.307500	3.472875	3.646519	3.828845	3	4.020287	4.221301	4.432366	4.653985	4.886684
4.200000	4.410000	4.630500	4.862025	5.105126	4	5.360383	5.628402	5.909822	6.205313	6.515579
5.250000	5.512500	5.788125	6.077531	6.381408	5	6.700478	7.035502	7.387277	7.756641	8.144473
6.300000	6.615000	6.945750	7.293037	7.657689	6	8.040574	8.442603	8.864733	9.307969	9.773368
7.350000	7.717500	8.103375	8.508544	8.933971	7	9.380669	9.849703	10.342188	10.859298	11.402262
8.400000	8.820000	9.261000	9.724050	10.210252	8	10.720765	11.256803	11.819644	12.410626	13.031157
9.450000	9.922500	10.418625	10.939556	11.486534	9	12.060861	12.663904	13.297099	13.961954	14.660052
11 Half years	12 Half years	13 Half years	14 Half years	15 Half years	If Principal is	16 Half years	17 Half years	18 Half years	19 Half years	20 Half years
Rupees	Rupees	Rupees	Rupees	Rupees		Rupees	Rupees	Rupees	Rupees	Rupees
1.710339	1.795856	1.885649	1.979932	2.078928	Rs. 1	2.182875	2.292018	2.406619	2.526950	2.653298
3.420679	3.591713	3.771298	3.959863	4.157856	2	4.365749	4.584037	4.813238	5.053900	5.306595
5.131018	5.387569	5.656947	5.939795	6.236785	3	6.548624	6.876055	7.219858	7.580851	7.959893
6.841357	7.183425	7.542597	7.919726	8.315713	4	8.731498	9.168073	9.626477	10.107801	10.613191
8.551697	8.979282	9.428246	9.899658	10.394641	5	10.914373	11.460092	12.033096	12.634751	13.266489
10.262036	10.775138	11.313895	11.879590	12.473569	6	13.097248	13.752110	14.439715	15.161701	15.919786
11.972376	12.570994	13.199544	13.859521	14.552497	7	15.280122	16.044128	16.846335	17.688651	18.573084
13.682715	14.366851	15.085193	15.839453	16.631425	8	17.462997	18.336147	19.252954	20.215602	21.226382
15.393054	16.162707	16.970842	17.819384	18.710354	9	19.645871	20.628165	21.659573	22.742552	23.879679
21 Half years	22 Half years	23 Half years	24 Half years	25 Half years	If Principal is	26 Half years	27 Half years	28 Half years	29 Half years	30 Half years
Rupees	Rupees	Rupees	Rupees	Rupees		Rupees	Rupees	Rupees	Rupees	Rupees
2.785963	2.925261	3.071524	3.225100	3.386355	Rs. 1	3.555673	3.733456	3.920129	4.116136	4.321942
5.571925	5.850521	6.143048	6.450200	6.772710	2	7.111345	7.466913	7.840258	8.232271	8.643885
8.357888	8.775782	9.214571	9.675300	10.159065	3	10.667018	11.200369	11.760387	12.348407	12.965827
11.143850	11.701043	12.286095	12.900400	13.545420	4	14.222691	14.933825	15.680517	16.464542	17.287769
13.929813	14.626304	15.357619	16.125500	16.931775	5	17.778363	18.667282	19.600646	20.580678	21.609712
16.715776	17.551564	18.429143	19.350600	20.318130	6	21.334036	22.400738	23.520775	24.696814	25.931654
19.501738	20.476825	21.500666	22.575700	23.704485	7	24.889709	26.134194	27.440904	28.812949	30.253597
22.287701	23.402086	24.572190	25.800800	27.090840	8	28.445382	29.867651	31.361033	32.929085	34.575539
25.073663	26.327346	27.643714	29.025899	30.477194	9	32.001054	33.611107	35.281162	37.045220	38.897481
31 Half years	32 Half years	33 Half years	34 Half years	35 Half years	If Principal is	36 Half years	37 Half years	38 Half years	39 Half years	40 Half years
Rupees	Rupees	Rupees	Rupees	Rupees		Rupees	Rupees	Rupees	Rupees	Rupees
4.538039	4.764941	5.003189	5.253348	5.516015	Rs. 1	5.791816	6.081407	6.385477	6.704751	7.039989
9.076079	9.529883	10.006377	10.506696	11.032031	2	11.583632	12.162814	12.770955	13.409502	14.079977
13.614118	14.294824	15.009566	15.760044	16.548046	3	17.375448	18.244221	19.156432	20.114253	21.119966
18.152158	19.059760	20.012754	21.013392	22.064061	4	23.167265	24.325628	25.541909	26.819005	28.159955
22.690197	23.824707	25.015943	26.266740	27.580077	5	28.959081	30.407035	31.927386	33.523756	35.199944
27.228237	28.589649	30.019131	31.520088	33.096092	6	34.750897	36.488442	38.312864	40.228507	42.239932
31.766276	33.354590	35.022320	36.773436	38.612108	7	40.542713	42.569849	44.698341	46.933258	49.279921
36.304316	38.119532	40.025508	42.026784	44.128123	8	46.334529	48.651256	51.083818	53.638009	56.319910
40.842355	42.884473	45.028697	47.280132	49.644138	9	52.126345	54.732662	57.469296	60.342760	63.359898

Table C

COMPOUND INTEREST TABLES

Showing growth of Principal Compounded QUARTERLY

8% For 91 days and above but less than one year.

If Principal is	1 Quarter	2 Quarters	3 Quarters
Ra. 100	102.0000	104.0400	106.1208
200	204.0000	208.0800	212.2416
300	306.0000	312.1200	318.3624
400	408.0000	416.1600	424.4832
500	510.0000	520.2000	530.6040
600	612.0000	624.2400	636.7248
700	714.0000	728.2800	742.8456
800	816.0000	832.3200	848.9664
900	918.0000	936.3600	955.0872

9% For 1 year and above but less than 2 years.

If Principal is	1 Quarter	2 Quarters	3 Quarters	4 Quarters	5 Quarters	6 Quarters	7 Quarters
Ra. 100	102.2500	104.5506	106.9030	109.3083	111.7678	114.2825	116.8539
200	204.5000	209.1013	213.8060	218.6167	223.5355	228.5651	233.7078
300	306.7500	313.6519	320.7090	327.9250	335.3033	342.8476	350.5617
400	409.0000	418.2025	427.6121	437.2333	447.0711	457.1302	467.4156
500	511.2500	522.7531	534.5151	546.5417	558.8388	571.4127	584.2695
600	613.5000	627.3038	641.4181	655.8580	670.6066	685.6953	701.1234
700	715.7500	731.8544	748.3211	765.1583	782.3744	799.9778	817.9773
800	818.4000	836.4050	855.2241	874.4667	894.1422	914.2604	934.8312
900	920.2500	940.9556	962.1271	983.7750	1005.9099	1028.5429	1051.6851

10% For 2 years and above.

If Principal is	1 Quarter	2 Quarters	3 Quarters	4 Quarters	5 Quarters	6 Quarters	7 Quarters	8 Quarters	9 Quarters	10 Quarters
Ra. 100	102.5000	105.0625	107.6891	110.3813	113.1408	115.9693	118.8686	121.8403	124.8863	128.0085
200	205.0000	210.1250	215.3781	220.7625	226.2816	231.9387	237.7372	243.6805	249.7726	256.0169
300	307.5000	315.1875	323.0672	331.1439	339.4225	347.9080	356.6057	365.5209	374.6589	384.0254
400	410.0000	420.2500	430.7563	441.5252	452.5632	463.8774	475.4743	487.3612	499.5452	512.0338
500	512.5000	525.3125	538.4453	551.9064	565.7041	579.8467	594.3429	609.2014	624.4315	640.0423
600	615.0000	630.3750	646.1344	662.2877	678.8449	695.8161	713.2115	731.0417	749.3178	768.0507
700	717.5000	735.4375	753.8234	772.6690	791.9857	811.7854	832.0800	852.8820	874.2041	896.0592
800	820.0000	840.5000	861.5125	883.0503	905.1265	927.7547	950.9486	974.7273	999.0904	1024.0676
900	922.5000	945.5625	969.2016	993.4316	1018.2674	1043.7241	1069.8172	1096.5626	1123.9767	1152.0761
If Principal is	11 Quarters	12 Quarters	13 Quarters	14 Quarters	15 Quarters	16 Quarters	17 Quarters	18 Quarters	19 Quarters	
Ra. 100	131.2087	134.4889	137.8511	141.2974	144.8298	148.4506	152.1618	155.9659	159.8650	
200	262.4173	268.9778	275.7022	282.5948	289.6596	296.9011	304.3237	311.9317	319.7300	
300	393.6260	403.4666	413.5533	423.8921	434.4894	445.3517	456.4856	467.8976	479.5951	
400	524.8347	537.9555	551.4044	565.1895	579.3193	593.8022	608.6473	623.8635	639.4601	
500	656.0433	672.4444	689.2555	706.4869	724.1491	742.2528	760.8091	779.8294	799.3251	
600	787.2520	806.9333	827.1066	847.7843	868.9789	890.7034	912.9710	935.7952	959.1901	
700	918.4607	941.4222	964.9577	989.0817	1013.8087	1039.1539	1065.1328	1091.7611	1119.0551	
800	1049.6693	1075.9111	1102.8088	1130.3791	1158.6385	1187.6045	1217.2946	1247.7270	1278.9201	
900	1180.8780	1210.3999	1240.6599	1271.6764	1303.4683	1336.0551	1369.4564	1403.6928	1438.7852	

Example 3 : Jagjeet saves Rs 50 from her income every month. She wants to put it in a recurring deposit so that she may get at least Rs 10000 on maturity. Calculate the minimum number of years for which she should make the deposit.

Solution : We examine the row headed by 50 in Table A. The amount which is nearest (and above) Rs 10000 is Rs 10278.50. This amount appears in the column headed by 120. Hence, she should make the deposit for 120 months or 10 years.

Example 4 : Anand invested a sum of Rs 50000 in a fixed deposit scheme for 19 years. If the rate of interest is 10% per annum compounded half yearly, find the amount that Anand would get after 19 years.

Solution : For the solution, we shall use Table B. In 19 years, i.e., in 38 half years, the principal of Rs 5 amounts to Rs 31.927386. Therefore, the principal of Rs 50000 in 19 years will amount to Rs 319273.86.

Example 5 : Kalidas deposited with a bank a sum of Rs 89500 for $1\frac{1}{2}$ years. If the rate of interest be 9% compounded quarterly, find the amount at the end of $1\frac{1}{2}$ years.

Solution : From Table C, we find that the amount of Rs 800 at 9% interest after 6 quarters ($= 1\frac{1}{2}$ years) is Rs 914.2604.

$$\begin{aligned}\text{Hence, the amount of Rs 80000} &= \text{Rs } (914.2604 \times 100) \\ &= \text{Rs } 91426.04\end{aligned}$$

$$\begin{aligned}\text{Similarly, the amount of Rs 9000} &= \text{Rs } (1028.5429 \times 10) \\ &= \text{Rs } 10285.43 \text{ and}\end{aligned}$$

$$\text{the amount of Rs 500} = \text{Rs } 571.41$$

Adding we get the amount of Rs 89500 as Rs 102282.88.

EXERCISE 8(a)

1. If Gulzari saves Rs 70 every month and puts it in a recurring deposit scheme for 96 months, what is the amount that he gets on maturity?
2. Manisha saves Rs 40 every month and puts it in the recurring deposit scheme for 84 months. After two years, her savings increased and she decided to put Rs 50 in another recurring deposit for 60 months.

Calculate the total amount that she gets on the maturity of the two schemes.

3. Ganapathi needs Rs 6000 after 7 years. Find the minimum amount of the monthly instalment for a recurring deposit that will serve his purpose.
4. Asha wants to save atleast Rs 15000 by saving Rs 90 a month in a recurring deposit scheme. Find the minimum number of instalments needed for the purpose.
5. Kushal deposited Rs 5680 with a finance company which pays half yearly interest at the rate of 10% per annum. Find the amount that he gets after $1\frac{1}{2}$ years.
6. Shahabuddin borrowed Rs 90000 for 9 months. If the rate of interest be 8% per annum compounded quarterly, find the interest that he pays at the end of 9 months. *3 months*
7. Subrahmanyam deposited Rs 9600 with a society which pays interest at the rate of 9% per annum. If the interest is compounded quarterly, what is the amount that he receives after 1 year?
8. Find the compound interest of Rs 5700 for $4\frac{1}{2}$ years at the rate of 10% per annum compounded quarterly.

Things to Remember

1. Banks are institutions which take deposits, lend money and help in transfer of money.
2. Banks operate different types of Accounts. Some such accounts are :
 - (a) Savings bank account
 - (b) Current account
 - (c) Fixed deposit account
 - (d) Recurring (cumulative) deposit account
3. Money is deposited in a bank through a pay-in-slip.
4. Money is withdrawn from a bank through a withdrawal form or a cheque.

5. Cheques may be of different types like :

- (a) Bearer cheque
- (b) Order cheque
- (c) Crossed cheque

CHAPTER NINE

Shares and Debentures

IN THIS CHAPTER we are going to talk about shares and debentures. Companies raise their capital through shares and may borrow money through debentures. Public invest money in these shares and debentures and earn dividend (profit) and interest on them respectively.

9.1 Joint Stock Company

Sometimes a group of persons intending to start a business or an industry are unable to arrange for the necessary finances on their own. What do they do then? In such cases, these persons prepare, with the help of experts, a detailed plan of the project and frame rules and regulations regarding its functioning. These are then registered under the Indian Companies Act(1956) and thus a *Joint Stock Company* comes into existence. A Joint Stock Company may raise the required capital through the public issue of shares.

9.2 Shares

Let us see how a joint stock company raises a capital, of Rs 5 crores say. It divides this amount in terms of a convenient unit called a *share*. Generally, the value of such a unit or share is Rs 10. Thus, the capital of Rs 5 crores will consist of 50 lakh shares of Rs 10 each. If, however, the value of a share is taken to be Rs 100, then the capital of Rs 5 crores will consists of 5 lakh shares of Rs 100 each. The general public is then invited to subscribe to the share capital of the company. Some people who accept the terms and conditions of the company and consider the investment profitable, apply for these shares. When one is allotted shares by the company and has paid the money prescribed by the company for the shares, the company issues to him a certificate called, *Share Certificate*. This is to show that he owns those shares. He then becomes a *share-holder* of the company.

9.3 Dividend

When a company makes a profit during a financial year, a part of the profit is kept in reserve fund, depreciation account etc. as also used for payment of Government taxes. The rest may be distributed at the end of the year amongst the share-holders in proportion to the shares held by them. This distributed profit is known as dividend. Dividend may be given as *per share* or as a *per cent*. Thus, we may say a 'dividend of Rs 5 per share'. This means a person having 10 shares will get Rs 50 as dividend. A '20% dividend' means that on a share of Rs 100, the share-holder gets Rs 20; on a share of Rs 10, one gets Rs 2 and so on.

9.4 Brokerage

A share like any other commodity can be sold and purchased in a market, called stock exchange. The sale and purchase of shares is, generally, done through agents called share-brokers or simply *brokers*. Brokers charge commission from buyers as well as from sellers. The broker's commission is called *brokerage*.

9.5 Values of a Share

The value for which a share is issued by the company is called the *nominal value*, *face value* or *par value* of the share. This value is printed on the share certificate. However, when we sell a share in the market, it may fetch a different value. This value is called the *market value* of the share. If the market value of a share is more than its nominal value, the share is said to be *above par* or *at a premium*. In case the market value is less than the face value, the share is said to be *below par* or *at a discount*. If the market value is the same as the face value, then the share is said to be *at par*. While purchasing a share from the market, a buyer has to pay market value to the share-holder and brokerage to the broker. The seller of the shares receives the market value but has to pay brokerage to the broker. Brokerage is calculated on the market value of the share. We illustrate the points discussed above with the help of the following examples:

Example 1 : Find what a buyer would have to pay for 500 shares of Rs 10 each quoted at Rs 74. What would be the gain to the share-holder, if he had purchased the shares at par ?

Solution :

The face value of 1 share = Rs 10

The market value of 1 share = Rs 74

The amount paid by the buyer for 500 shares = Rs (74 × 500)
= Rs 37000

$$\begin{aligned}\text{The gain from 1 share} &= \text{Rs } (74 - 10) = \text{Rs } 64 \\ \therefore \text{The gain from 500 shares} &= \text{Rs } (64 \times 500) \\ &= \text{Rs } 32000\end{aligned}$$

Example 2 : Swarna invested Rs 2592 when she bought shares of a company at Rs 108 each. The face value of a share was Rs 100. The company paid $12\frac{1}{2}\%$ dividend at the end of the year. Find the dividend earned by Swarna.

Solution :

$$\text{Market value of one share} = \text{Rs } 108$$

$$\text{The amount invested} = \text{Rs } 2592$$

$$\text{Therefore, the number of shares bought} = \frac{2592}{108} = 24$$

$$\text{The face value of 24 shares} = \text{Rs } (100 \times 24)$$

$$\begin{aligned}\text{The dividend received} &= \text{Rs } \left(100 \times 24 \times \frac{25}{2} \times \frac{1}{100} \right) \\ &= \text{Rs } 300\end{aligned}$$

Example 3 : Ankur invested Rs 4444 in the shares of face value Rs 100 each of a company. At the end of the year, the company declared dividend at 15% which gave Ankur an income of Rs 600. At what price was the share quoted if the brokerage was 1%?

Solution :

$$\text{Rs } 600 \text{ is the income when the investment} = \text{Rs } 4444$$

$$\therefore \text{Rs } 1 \text{ is the income when the investment} = \text{Rs } \frac{4444}{600}$$

$$\therefore \text{Rs } 15 \text{ will be the income on an investment of Rs } \frac{4444}{600} \times 15 = \text{Rs } 111.1$$

Since the brokerage is 1%, a share of market value Rs 100 costs Ankur Rs 101. Thus, if the investment is Rs 101, then the market value = Rs 100

$$\therefore \text{if the investment is Rs } 1, \text{ then the market value} = \text{Rs } \frac{100}{101}$$

$$\begin{aligned}\therefore \text{if the investment is Rs } 111.1, \text{ then the market value} &= \text{Rs } \left(\frac{100}{101} \times 111.1 \right) \\ &= \text{Rs } 110\end{aligned}$$

Example 4 : Hari bought 300 shares of a company quoted at Rs 450. If the brokerage be 1%, find the amount spent by Hari on this purchase.

Solution :

The cost of one share = Rs 450

∴ The cost of 300 shares = Rs (450 × 300)
= Rs 135000

Also, the brokerage is 1% which is also to be paid by Hari. The brokerage on 300 shares is Rs $\left(\frac{1}{100} \times 135000\right)$ = Rs 1350

Therefore, the total amount spent by Hari = Rs (135000 + 1350)
= Rs 136350

EXERCISE 9(a)

[Note : Unless otherwise stated, no brokerage is involved.]

1. A company issued shares at 20% premium. Ravi applied for 1000 shares but was allotted 500 shares of this company. Find his investment, if the face value of a share is Rs 100.
2. Mohan invested Rs 11000 and purchased 1000 shares of face value Rs 10 each. How much above par is the market value of these shares? If the dividend per share is 50 paise, find Mohan's income from these shares.
3. Find the income derived from 600 shares, if the face value of one share is Rs 10, and the dividend is 16%.
4. The shares of a company are quoted at Rs 80. If the nominal value of a share is Rs 10 and if the company declares 15% dividend, what does it cost Satyajit to buy 4000 shares and what dividend does he get?
5. Gyanchand sold 500 shares of a company with face value of a share as Rs 100 and market value as Rs 740. What was his gain if he had purchased the shares at 100% premium?
6. Zaved invested Rs 5500 in the shares of a company. At the end of the year, the company declared 20% dividend which gave Zaved an income of Rs 500. At what premium were the shares purchased?
7. Abdulla purchased 300 shares of the face value of Rs 100 each from the market at Rs 800 per share. If the company paid a dividend of 40%, find Abdulla's earning per cent on this investment.

8. How many shares of market value Rs 12.50 each can be purchased for Rs 12625, if the brokerage paid is 1%?
9. Satya invested Rs 2750 when she bought 100 rupee shares of a company available for Rs 110 each. If the company paid 15% dividend during the year after this purchase, find Satya's income from this dividend.
10. 100 rupee share of a company is quoted at Rs 899 in the market. How much does Venkat pay to purchase 500 such shares, if the brokerage paid by him is 1%? What is his gain per cent on this investment, if the company pays a dividend of 25%?

9.6 Debentures

When business grows, a company may go in for further expansion and diversification of its activities. For this, the company may need more money. For this additional money, the company may not issue new shares. It may, instead, like to borrow this money from the shareholders/public for a fixed period at a fixed rate of interest. In such a case, the company issues *debentures*. If a company wants to borrow Rs 20 lakhs, it may issue 10000 debentures of value Rs 200 each. When the fixed period is over, the company repays the debenture money to its holders. The interest on debentures is paid generally six-monthly.

Like shares, the market value of debentures also keeps on changing. However, the interest on a debenture is always calculated on its face value. The brokerage involved in the sale or purchase of debentures is calculated on their market value.

The main difference between shares and debentures is that shares form a part of the capital of the company, whereas debentures are a debt. Shareholders receive dividend commensurate with profit earned by the company. This dividend may vary from year to year. On the other hand, debenture-holders receive interest six-monthly (or as specified earlier by the company) at a fixed rate. This interest they receive irrespective of the profit or loss of the company.

Example 1 : Find the income per cent on 4% debentures of face value Rs 100 and available at Rs 80 each.

Solution : Rs 80 cash buys a Rs 100 debenture which gives Rs 4 as interest.

Therefore, Rs 100 will give an interest of $\text{Rs } \frac{4 \times 100}{80} = \text{Rs } 5$

Hence, the income is 5%.

Example 2 : Sophia wants to secure an annual income of Rs 1500 by investing in 15% debentures of face value Rs 100 each and available for Rs 104 each. If the brokerage is 1%, what sum of money should she invest ?

Solution : Here debentures are being purchased so the brokerage will be added. To purchase one debenture, the amount to be invested = cost of the debenture + brokerage paid = Rs 104 + Rs 1.04 = Rs 105.04. This investment of Rs 105.04 brings an income of Rs 15.

Now Rs 15 is the income from an investment of Rs 105.04

$$\therefore \text{Rs } 1 \text{ is the income from an investment of Rs } \frac{105.04}{15}$$

$$\therefore \text{Rs } 1500 \text{ is the income from an investment of Rs } \frac{105.04 \times 1500}{15}$$

$$= \text{Rs } 10504$$

Example 3 : How much money will Nisha get by selling 20% debentures worth Rs 6000 at the rate of Rs 120 per debenture ? The face value of a debenture is Rs 100 and the brokerage is $1\frac{1}{2}\%$.

Solution : The selling price of a debenture worth Rs 100 = Rs 120

$$\text{Hence, the selling price of debentures worth Rs } 6000 = \text{Rs } \frac{120}{100} \times 6000$$

$$= \text{Rs } 7200$$

Now brokerage is $\frac{3}{2}\%$.

$$\text{Hence, the total brokerage} = \text{Rs } \frac{3}{2} \times \frac{7200}{100}$$

$$= \text{Rs } 108$$

$$\text{Thus, amount received by Nisha} = \text{Selling price} - \text{brokerage}$$

$$= \text{Rs } (7200 - 108)$$

$$= \text{Rs } 7092$$

$$\begin{array}{r} 50 \text{ Rs} \\ 25 \text{ Rs} \\ \hline 2000 \end{array}$$

EXERCISE 9(b)

1. Find the annual yield per cent on 15% debentures of face value Rs 100 each and available at Rs 80 each.
2. Suraj had 300 debentures of a company from which he received half-yearly interest of Rs 750. If the rate of interest be 10% per annum, find the face value of one debenture.
3. Ashu invested Rs 12000 in 11.5% debentures of the face value of Rs 100 each available at Rs 120. What is his annual income ?
4. Calculate the rate per cent that Chidambaram earns by investing in 8% debentures of the face value of Rs 100 each available at Rs 112.
5. Which is a better investment: 15% debentures at 8% premium or 14% debentures at 4% discount ?
6. Which is a better investment: 10% debentures at 10% discount or 20% debentures at 10% premium ?
7. Gauri sold 20% debentures worth Rs 7500 at 5% premium. Find the cash released from this sale, if the brokerage was 2%.
8. Sulabh buys 500 debentures of the face value of Rs 100 each at Rs 95 and sells the same when the price rises to Rs 98. If the brokerage be 2% on each transaction, what is the gain or loss of Sulabh?
9. Preeti invested a certain sum in $12\frac{1}{2}$ % debentures of the face value of Rs 100 each available at Rs 85, and earned an annual income of Rs 4000. Find the amount invested by her, if the brokerage is $1\frac{1}{2}$ %.
10. Shreekanth invested a part of Rs 5000 in $13\frac{1}{2}$ % debentures of the face value of Rs 100 each available at Rs 100. The rest he invested in Rs 100 shares available at par. At the end of the year, he received 20% dividend on his investment on shares. If his income from shares is equal to his income from his investment in debentures, find the amounts invested in shares and debentures.

Things to Remember.

1. The convenient unit (generally Rs 10) in which the capital of a joint stock company is divided, is called a *share*.
2. The value for which a share is issued by a company is called the *nominal value*, *face value* or *par value* of the share.
3. The value for which a share is available in the market is known as the *market value* of the share.
4. If the market value of a share is more than its face value, the share is said to be *above par*.
5. If the market value of a share is less than its face value, the share is said to be *below par*.
6. When the face value of a share is equal to its market value, the share is said to be *at par*.
7. That part of the profit of a company which is distributed amongst the share-holders is known as *dividend*.
8. *Debentures* are a debt of a company. A debenture-holder receives interest on the debentures at a fixed rate from the company.
9. Dividend on shares and interest on debentures is calculated on their face values.
10. Shares and debentures are, generally, sold or purchased in a market known as the *stock exchange* through authorised persons known as *share-brokers* (brokers). Broker's commission is known as the *brokerage*. Brokerage is always calculated at the market value of the shares/debentures.

Answers

REVIEW EXERCISE

1. (d)
2. (d) ; (b) and (c)
3. Numerators : 3, -5, 7, 6, 0, 9
Denominators : 7, 9, -11, 1, 1, 3
4. $\frac{2}{3}$, $\frac{-2}{-3}$
7. A rational number $\frac{p}{q}$ is said to be in the standard form if q is positive and p, q have no common divisor other than 1.
 $\frac{3}{7}$, $\frac{1}{14}$, $\frac{-3}{1}$, $\frac{-3}{7}$, $\frac{5}{9}$
8. $\frac{-3}{5}$
9. $7\frac{3}{4}$, $4\frac{1}{2}$, $8\frac{3}{5}$,
 $15\frac{1}{8}$, $-6\frac{10}{13}$, $-8\frac{3}{5}$
10. $\frac{19}{5}$, $\frac{20}{3}$, $\frac{-21}{4}$, $\frac{-23}{3}$
12. 7 and 8
4 and 5
8 and 9
15 and 16
-7 and -6
-9 and -8
- 3 and 4
6 and 7
-6 and -5
13. Equal : (a), (b), (c) and (e)
Not Equal : (d) and (f)
15. $\frac{5}{10}$, $\frac{8}{16}$, $\frac{13}{26}$, $\frac{51}{102}$, $\frac{2500}{5000}$; No
16. $\frac{10}{16}$, $\frac{35}{56}$, $\frac{80}{128}$, $\frac{1200}{1920}$, $\frac{5000}{8000}$; No
17. $\frac{17}{119}$ and $\frac{19}{95}$
18. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{5}$, $\frac{1}{4}$, $\frac{3}{2}$, $\frac{16}{3}$
19. Ascending Order : $\frac{-7}{11}$, $\frac{-5}{9}$,
 $\frac{0}{1}$, $\frac{3}{7}$, $\frac{9}{3}$, $\frac{6}{1}$
Descending Order : $6\frac{2}{3}$, $3\frac{4}{5}$,
 $-5\frac{1}{4}$, $-7\frac{2}{3}$
20. (a) $\frac{23}{12}$ (b) $\frac{15}{8}$ (c) $\frac{167}{60}$ (d) $\frac{67}{70}$
21. (a) 3 (b) $\frac{511}{144}$ (c) $\frac{85}{36}$ (d) $\frac{475}{336}$
22. $\frac{-2}{3}$, $\frac{-4}{5}$, $\frac{3}{5}$, $\frac{7}{8}$, $\frac{6}{7}$, 0
23. (a) $\frac{2}{5}$ (b) $\frac{2}{15}$ (c) $\frac{-19}{30}$
(d) $\frac{9}{5}$ (e) $\frac{-10}{7}$ (f) $\frac{-7}{9}$

24. (a) $\frac{13}{10}$ (b) $\frac{9}{14}$
 (c) $\frac{-11}{8}$ (d) $\frac{-11}{4}$
25. (a) $\frac{15}{2}$ (b) $\frac{-50}{3}$
 (c) $\frac{165}{2}$ (d) -152
26. (a) $\frac{7}{10}$ (b) $\frac{4}{11}$
 (c) $\frac{-7}{9}$ (d) $\frac{-2}{33}$
27. (a) $\frac{7}{8}$ (b) $\frac{-6}{7}$ (c) $\frac{-4}{7}$
 (d) $\frac{8}{7}$ (e) 0 (f) 0
28. (a) $\frac{19}{2}$ (b) $\frac{-5}{4}$
 (c) $\frac{110}{21}$ (d) $\frac{-7}{2}$
29. $\frac{1}{7}, \frac{13}{7}, \frac{9}{31}, \frac{5}{32}$
30. (a) $\frac{8}{7}$ (b) $\frac{-7}{3}$ (c) $\frac{4}{5}$
 (d) $\frac{-8}{5}$ (e) $\frac{6}{5}$ (f) 0
31. (a) $\frac{25}{242}$ (b) $\frac{1}{2}$ (c) $\frac{11}{7}$ (d) 0
33. 4, 23, $\frac{47}{13}$, $\frac{3}{10}$, $\frac{21}{43}$
34. 0.25, 0.5, 0.75, 1.5, 1.25,
 0.875, 0.6, $0.\bar{3}$, $0.\bar{4}$, $0.2\bar{1}$
35. $\frac{19}{9}$, $\frac{29}{9}$, $\frac{716}{99}$, $\frac{217}{30}$, $\frac{269}{33}$
36. Yes ; not same
37. (a) $\left(\frac{2}{3}\right)^2$ (b) $\left(\frac{3}{4}\right)^3$ (c) $\left(\frac{-5}{3}\right)^3$
 (d) $\left(\frac{-4}{3}\right)^5$ (e) $\left(\frac{16}{81}\right)^1$
38. $\frac{4}{49}$, $\frac{27}{64}$, $\frac{49}{36}$, $\frac{-27}{125}$
39. (a) $\left(\frac{2}{5}\right)^5$ (b) $\frac{3}{4}$ (c) $\left(\frac{3}{5}\right)^{-11}$ or $\left(\frac{5}{3}\right)^{11}$
 (d) 1 (e) 1 (f) $\left(\frac{4}{3}\right)^6$
40. 1.23456×10^2 , 2.345678×10^3 ,
 4.5678×10^{-1} , 7.6543×10^{-3}
41. 131.1, 65120.5,
 0.006123, 0.00074612
42. (a) Inverse (b) Direct
 (c) Direct (d) Inverse
 (e) Neither
43. $\frac{3}{4}$
44. $2\frac{2}{5}$ hours
45. $18\frac{3}{4}$ days
46. 49 days
47. 42 km
48. $4\frac{1}{2}$ hours
49. 7 km/hr
50. Rs 2375
51. 79.6%

52. Rs 1197
53. $16\frac{2}{3}\%$
54. Rs 700
55. Profit of Rs 4
56. Rs 2000; Rs 200
57. Rs 180; Rs 680
58. Rs 250
59. 5 years
60. $6\frac{1}{4}\%$
61. Rs 150
62. 12%
63. Rs 1693.62
64. Rs 378.90
65. 20%
66. Rs 352.80
67. QR ; $\angle C$
68. QR = 4 cm ; PQ = 5 cm
69. Yes
70. They are congruent
71. Yes
72. Not necessarily
77. $\triangle BCO$ and $\triangle DAO$
78. Yes
79. Yes
80. (6, 8, 10) , (14, 48, 50) and (12, 16, 20)
81. (a) True (b) True
(c) False; The point where the altitudes of a triangle meet is known as the orthocentre.
(d) False; The orthocentre of an acute triangle lies in the interior of the triangle.
(e) True
(f) False; An equilateral triangle has equal altitudes.
(g) False ; The orthocentre of an equilateral triangle coincides with the centroid.
82. (c) False; The point of concurrence of the medians is known as the centroid.
OR, The point of concurrence of the perpendicular bisectors of the sides is known as the circumcentre.
(d) False ; The centroid divides the medians in the ratio 2 : 1.
(f) False ; The centroid of an obtuse triangle lies in the interior of the triangle.
83. (a) Circumcentre
(b) An acute triangle

- (c) In the exterior of the triangle
 (d) At the middle point of the hypotenuse.
 (e) Not necessarily
 (f) The circle passing through all the vertices of the triangle
 (g) They all are the same; that is, they coincide.
 (h) Centre of the circle touching all the sides of the triangle.
 (i) Not necessarily
 (j) Incircle
 (k) CO bisects $\angle C$.
84. (a) 4, 4, 2
 (b) 360°
 (c) Diagonal ; Side
 (d) Trapezium
 (e) Parallelogram
 (f) Equal ; Angles
 (g) Bisect
 (h) Parallelogram
 (i) Equal
 (j) Rhombus
 (k) Perpendicular ; bisect
 (l) 90° each
 (m) Equal and also bisect each other at right angles
85. 24 cm^2
 86. 7 cm
 87. 3 cm
 88. 150 cm^2
 89. 46 cm
90. (a) $6xy^2$ (b) $75xy^2z$
 (c) $21a^3b^5c^2$ (d) $240 r^3s^4t^4$
91. (a) $3x^3y^3 + 4xy^2$
 (b) $x^4y^2 - xy^4$
 (c) $6x^5 + x^3y - 5xy^2$
 (d) $p^3q + 2p^2q^2 + pq^3$
92. (a) $x^2 + 2xy + y^2$
 (b) $x^2 - 2xy + y^2$
 (c) $x^2 - y^2$
93. $x^2 + 12xy + 36y^2$
 $4a^2 - 12ab^2 + 9b^4$
 $p^4 - 14p^2q^2 + 49q^4$
94. 2601, 39601, 40401
95. (a) 1, a, b, ab
 (b) 1, 4, s, x, r, $4sxr$, $4s$, xr , $4x$, sr , $4r$, sx , $4xr$, $4sr$, $4sx$, sxr
 (c) 1, 4, x, x^2 , y, z, $4x^2yz$, $4x^2$, yz , $4y$, x^2z , $4z$, x^2y , $4yz$, $4x^2z$, $4x^2y$, x^2yz , $4xyz$, $4x$, $4xy$, xy , xyz , xz , $4xz$
96. (a) xy (b) $6pq$ (c) xyz
97. (a) $2xyz(2x + 3yz^2)$
 (b) $abc(1 - 5bc^2)$
 (c) $-5abc^2(4c + 3ab)$
98. (a) $(a^2 + b^2)(x + y)$
 (b) $(a - b)(x + y)(x - y)$
99. (a) $(2x + 3y)^2$
 (b) $(a - 2b)^2$
 (c) $(8x + 9y)(8x - 9y)$

100. (a) $x = 3$ (b) $x = 12$
(c) $x = \frac{-10}{3}$

101. (a) $\frac{1}{2}$ (b) -30

EXERCISE 1(a)

- 1. 16 , 36
- 2. 256 , 1296
- 3. 100
- 4. 121
- 5. (a) 45 (b) 201 (c) 1101
- 6. (a) 4225 (b) 5625
- 7. (3, 4, 5) ; (6, 8, 10)
- 8. 20, 6, 42, 43
- 9.
$$\frac{(4444)^2}{1+2+3+4+3+2+1}$$
$$\frac{(55555)^2}{1+2+3+4+5+4+3+2+1}$$
$$\frac{(666666)^2}{1+2+3+4+5+6+5+4+3+2+1}$$
$$\frac{(7777777)^2}{1+2+3+4+5+6+7+6+5+4+3+2+1}$$
$$\frac{(88888888)^2}{1+2+3+4+5+6+7+8+7+6+5+4+3+2+1}$$
- 10. 6561, 6724, 6889, 7056 , 7225
7396 , 7569 , 7744 , 7921
- 11. (a) Yes (b) No

EXERCISE 1(b)

- 1. 5
- 2. 8
- 3. 9
- 4. 10
- 5. 13
- 6. 15
- 7. 20
- 8. 70
- 9. 39
- 10. 320

EXERCISE 1(c)

- 1. (a) 4 (b) 14 (c) 23
(d) 20 (e) 42 (f) 64
(g) 88 (h) 108 (i) 70
(j) 217 (k) 1000 (l) 546
- 2. 5 , 30
- 3. 3 , 56
- 4. 77
- 5. 480

EXERCISE 1(d)

- 1. $\frac{1}{2}$
- 2. $\frac{4}{5}$
- 3. 0.3

4. 0.02

5. $\frac{13}{17}$

6. $\frac{11}{100}$

7. 0.25

8. 0.18

EXERCISE 1(e)

1. $\frac{25}{36}$

2. $2\frac{1}{7}$

3. $1\frac{9}{14}$

4. $4\frac{9}{11}$

5. $7\frac{11}{46}$

6. $8\frac{5}{7}$

7. 2.403

8. 0.0231

9. 897

10. $10\frac{1}{20}$ m

EXERCISE 1(f)

1. 1, 2, 2, 3, 3, 5.

2. 48

3. 67

4. 59

5. 99

6. 210

7. 165

8. 234

9. 222

10. 316

11. 625

12. 345

13. 440

14. 1111

15. 2222

16. 4607

17. 9070

18. 8027

19. 7906

20. 62573

21. '56804

22. 80047

23. 57

24. 110

25. 40

26. 25

27. 1024

28. 998001

29. 90 soldiers

EXERCISE 1(g)

- 1. $\frac{19}{25}$
- 2. $2\frac{2}{5}$
- 3. $5\frac{6}{7}$
- 4. $9\frac{2}{11}$
- 5. $4\frac{23}{27}$
- 6. $4\frac{8}{13}$
- 7. $3\frac{4}{15}$
- 8. $5\frac{35}{38}$
- 9. $18\frac{3}{13}$
- 10. 2.7
- 11. 4.1
- 12. 3.05
- 13. 9.21
- 14. 12.25
- 15. 21.85
- 16. 15.02
- 17. 0.0091
- 18. 0.231
- 19. 501.95
- 20. 15.093
- 21. 16.02 m

EXERCISE 1(h)

- 1. 1.304
- 2. 4.806
- 3. 2.236
- 4. 15.415
- 5. 0.949
- 6. 4.472
- 7. 0.316
- 8. 0.126
- 9. 0.025
- 10. 2.646
- 11. 0.935
- 12. 8.124
- 13. 3.914
- 14. 1.581
- 15. 0.645
- 16. 1.443
- 17. 16.960
- 18. 19.165
- 19. 3.4651
- 20. 3.31662

EXERCISE 1(i)

- 1. 2.646
- 2. 3.873
- 3. 9.055
- 4. 0.346

5. 23.24
6. 93.27
7. 36.22
8. 3.635
9. 7.458
10. 8.473
11. 64.75
12. 70.39
13. 160.39
14. 0.773
15. 18.028 m
16. 15.49 m

EXERCISE 1(i)

1. $.5$, $0.\overline{3}$, $0.\overline{4}$,
 0.615384 , 14.0625
2. $\frac{1406}{25}$

EXERCISE 2(a)

1. 343 , 1728 , 9261,
1000000 , 27543608
6. 64 , 216 , 1728
7. 7
8. 5
9. 2197 m³

EXERCISE 2(b)

1. (i) 7 (ii) 10 (iii) 14
(iv) 42 (v) 100
2. (i) 512 (ii) 364 (iii) 420
3. 5
4. 1183 , 546
5. 36 , 9
6. 8 m

EXERCISE 2(c)

1. (a) and (b)
2. (a) -5 (b) -18
(c) -26 (d) -140

EXERCISE 2(d)

1. 8
2. -72
3. -42
4. 75
5. 225
6. -77
7. -78
8. -180

EXERCISE 2(e)

1. (a) $\frac{343}{729}$ (b) $-\frac{512}{1331}$

$$(c) \frac{1728}{343} \quad (d) -\frac{2197}{512}$$

$$(e) \frac{2197}{125} \quad (f) \frac{2197}{64}$$

$$2. (a) 0.027 \quad (b) 3.375$$

$$(c) 0.000512 \quad (d) 9.261$$

$$3. (a) \text{ and } (c)$$

$$4. (a) \frac{2}{5} \quad (b) \frac{-4}{11}$$

$$(c) \frac{-3}{16} \quad (d) \frac{13}{21}$$

$$5. 3.2 \text{ m}$$

EXERCISE 2(f)

$$1. 2.351$$

$$2. 6.300$$

$$3. 10.32$$

$$4. 0.341$$

$$5. 0.07416$$

$$6. 17.2232$$

$$7. 25.742$$

$$8. 21.40$$

$$9. 67.63$$

$$10. 4.4123$$

$$11. 9.84$$

$$12. 17.459$$

$$13. 6.503 \text{ m}$$

EXERCISE 3(a)

$$1. 5^{\frac{1}{2}}, 7^{\frac{1}{3}}, (1100)^{\frac{1}{9}}, \left(\frac{3}{4}\right)^{\frac{1}{4}}, \left(\frac{61}{1123}\right)^{\frac{1}{8}}$$

$$2. \sqrt{16}, \sqrt[3]{125}, \sqrt[9]{\frac{6}{17}}$$

$$\sqrt[11]{\frac{11}{17}}, \sqrt[17]{\frac{61}{325}}$$

EXERCISE 3(b)

$$1. 2$$

$$2. 9$$

$$3. 32$$

$$4. \frac{1}{8}$$

$$5. \frac{1}{36}$$

$$6. \frac{1}{32768}$$

$$7. 25$$

$$8. 49$$

$$9. 9$$

$$10. \frac{1}{4}$$

EXERCISE 3(c)

$$1. (a) (37)^{\frac{1}{4}} \quad (b) (27)^{\frac{1}{5}}$$

$$(c) (29)^{\frac{2}{7}} \quad (d) \left(\frac{8}{9}\right)^{\frac{1}{6}}$$

EXERCISE 4(a)

- (e) $\left(\frac{2}{3}\right)^{\frac{1}{4}}$ (f) 2^{-2}
2. (a) $\sqrt[4]{5}$ (b) $\sqrt[3]{21^2}$
 (c) $\sqrt[6]{2^5}$ (d) $\sqrt[9]{\frac{5}{17}}$
- (e) $\sqrt[5]{\left(\frac{17}{21}\right)^2}$ (f) $\sqrt[7]{-215}$
3. (a) 2 (b) 24 (c) $\frac{1}{8}$
 (d) $\frac{1}{1000}$ (e) 3
4. (a) 49 (b) 121
 (c) 18 (d) $\frac{1}{3}$
5. (a) 13 (b) 100
 (c) 12100 (d) 1
6. (a) 24 (b) $\frac{2}{9}$
 (c) $\frac{1}{25}$ (d) $\frac{1}{8}$
7. (a) 0.008 (b) 0.04
 (c) 0.00032 (d) 15.625
8. (a) 72 (b) -2
 (c) 328 (d) -1260
9. (a) 5 (b) 13
 (c) 17 (d) 6
10. (a) $\frac{1}{x^{12}}$ (b) $\frac{4}{x}$
 (c) $\frac{1}{x^7 y^7}$ (d) $\frac{1}{x}$

1. (a) $x^2 + 11x + 18$
 (b) $y^2 + 12y + 35$
 (c) $x^2 + 6x - 16$
 (d) $z^2 - 8z + 15$
 (e) $p^2 - 7p - 18$
 (f) $x^2 + \frac{25}{12}x + 1$
 (g) $y^2 - 11y + 18$
 (h) $z^4 - z^2 - 6$
2. (a) 10812
 (b) 9888
 (c) 9215
 (d) 2915
 (e) 1224

EXERCISE 4(b)

1. $x^2 + 4y^2 + 9z^2 + 4xy + 12yz + 6xz$
 2. $x^2 + y^2 + 4z^2 + 2xy - 4yz - 4xz$
 3. $4p^2 + 4q^2 + 9r^2 + 8pq - 12qr - 12pr$
 4. $p^2 + 9q^2 + 4z^2 - 6pq + 12qz - 4pz$
 5. $x^2 + 25y^2 + 4z^2 - 10xy - 20yz + 4xz$
 6. $81x^2 + 4y^2 + 9z^2 - 36xy + 12yz - 54xz$
 7. $9m^2 + 25n^2 + 4p^2 + 30mn - 20np - 12mp$
 8. $9x^2 + \frac{1}{4}p^2 + 4q^2 - 3xp - 2pq + 12xq$
 9. $25x^4 + y^2 + z^2 + 10x^2y + 2yz + 10x^2z$
 10. (a) 4, 9, 25, 12, 30, 20
 (b) 4, 2, 4, 4
 11. (a) $2x^2 + 2y^2 + 2z^2 + 4xy$
 (b) $8px - 8cx$

EXERCISE 4(c)

- $x^3 + 27y^3 + 9x^2y + 27xy^2$
 - $27x^3 - 8y^3 - 54x^2y + 36xy^2$
 - $p^3x^3 + 8z^3 + 6p^2x^2z + 12pxz^2$
 - $x^6 + y^3 + 3x^4y + 3x^2y^2$
 - $8x^3 - y^6 - 12x^2y^2 + 6xy^4$
 - $343x^3 + 729y^3 + 1323x^2y + 1701xy^2$
 - $8z^3 - 343y^3 - 84yz^2 + 294y^2z$
 - $\frac{8}{27}x^3 - \frac{125}{27}z^3 - \frac{20}{9}x^2z + \frac{50}{9}xz^2$
- 728
 - $\frac{7440}{27}$
- 224
 - $\frac{5050}{729}$
- 15616
 - $\frac{27027}{125}$
- $16x^3 + 108xp^2$
 - $16p^3 + 12x^2p$
 - $16x^3 + 12xz^2$
 - $-250n^3 - 1470t^2n$
 - $\frac{2y^3}{125} + \frac{2}{15}x^2y$
- 157625
 - 970299
 - 28787625
 - 1015075125
 - 97002999
 - 217081801
 - 14921799
 - 65450827
 - 157.625
 - 970.299
- $x^3n^3, 9, 27, 27$
 - $1, 3x^2, y^6$
 - $\frac{1}{2}y, \frac{8}{7}, \frac{72}{49}, \frac{216}{343}$
 - $8, 36, 54$

EXERCISE 4(d)

- $x^3 + 8$
 - $1 - x^3$
 - $27x^3 + 125$
 - $343y^3 - 125z^3$
 - $\frac{8x^3}{125} - \frac{27y^3}{343}$
 - $.729x^3 + .343y^3$
- $4t^2, 9$
 - $9z^2, 4, 8$
 - $1, x$
 - x^4

EXERCISE 4(e)

- $(x + 4)(x + 5)$
 - $(x - 4)(x - 2)$
 - $(z + 4)(z - 3)$
 - $(p + 9q)(p - 4q)$
 - $(m - 5)(m + 3)$
 - $(m + 9n)(m + 2n)$
- $(x + 2y - z)^2$
 - $(2p + 3q + 2r)^2$
 - $(2x - y - 3z)^2$
 - $(p + \frac{q}{2} + 1)^2$
 - $(\sqrt{2}x + 2y - \sqrt{3}z)^2$
- $(3x - 2y)^3$
 - $(x + 4y)^3$
 - $(2y - 5z)^3$
 - $(x + 5z)^3$
- $(p + 3)(p^2 - 3p + 9)$
 - $(y + 5)(y^2 - 5y + 25)$
 - $(1 - 3z)(1 + 3z + 9z^2)$

(d) $(2xy + 3z)(4x^2y^2 - 6xyz + 9z^2)$

(e) $(4x - y)(16x^2 + 4xy + y^2)$

(f) $(m - 3n)(m^2 + 3mn + 9n^2)$

(g) $\left(\frac{1}{6}p - 2q\right)\left(\frac{1}{36}p^2 + \frac{pq}{3} + 4q^2\right)$

5. (a) $10xy(y - x)(y^2 + x^2 + yx)$

(b) $2x^3y(3x + y)(9x^2 - 3xy + y^2)$

(c) $4(2p + 3q)(4p^2 - 6pq + 9q^2)$

(d) $(p - 2q)(p + 2q)(p^2 + 4q^2 + 2pq)$
 $(p^2 + 4q^2 - 2pq)$

(e) $(p - 10q)(p^2 + 52q^2 + 4pq)$

(f) $(3x + m - 3)(9x^2 + m^2 - 18x$
 $- 3mx + 3m + 9)$

EXERCISE 4(f)

1. (a) $(p + 2q + 4r)(p^2 + 4q^2 + 16r^2$
 $- 2pq - 8qr - 4pr)$

(b) $(2x - 3y + z)(4x^2 + 9y^2 + z^2$
 $+ 6xy + 3yz - 2xz)$

(c) $(l + m - n)(l^2 + m^2 + n^2 - lm$
 $+ mn + ln)$

(d) $(-3x + y - z)(9x^2 + y^2 + z^2$
 $+ 3xy + yz - 3xz)$

(e) $(x - 2y - 4z)(x^2 + 4y^2 + 16z^2$
 $+ 2xy - 8yz + 4xz)$

(f) $\left(\frac{1}{3}x - y + 5z\right)\left(\frac{1}{9}x^2 + y^2 + 25z^2$
 $+ \frac{1}{3}xy + 5yz - \frac{5}{3}xz\right)$

2. (a) $9(3x - 5y)(5y - 9z)(3z - x)$

(b) $3(p - 3q)(3q - 7r)(7r - p)$

(c) $3(5x - 6p)(7z - 5x)(6p - 7z)$

(d) $3\left(\frac{1}{2}x - 3y\right)(3y - \sqrt{3}z)\left(\sqrt{3}z - \frac{1}{2}x\right)$

(e) $3pqr(q - r)(r - p)(p - q)$

3. (a) 123750 (b) -310764

(c) -498.33 (d) 14.256

EXERCISE 5(a)

1. (a) $2xz$ (b) $3n$

(c) $x - y$ (d) $3x + y$

2. (a) $4my$ (b) $x + 2$

(c) $4m + 3n$ (d) $5x + 4$

(e) $3x - 4y$

(f) $36z^2 + 42zp + 49p^2$

EXERCISE 5(b)

1. (a) 3 (b) 4 (c) 2

(d) 7 (e) 2 (f) 4

(g) 0

2. (a) $3m^3 + 4m^2 - 2$

(b) $x + 4$

(c) $2y + \frac{1}{2}$

(d) $-8z^2 - 12z + 2$

(e) $3m^2 - 2m + 9$

(f) $3y^2 + 3y + 2$

(g) $y^2 + 5y + 3$

3. (a) $2x + 3$, Remainder: -3.

(b) $5z^2 + \frac{10}{3}z + 11$, Remainder: 34

(c) $3y^3 - 5y + \frac{3}{2}$

(d) $-4x^3 + 2x^2 - 8x + 30$,
Remainder: -285

(e) $5y^3 - 2y^2 + \frac{5}{3}y$,

Remainder: 6

4. (a) Yes (b) No (c) No
(d) Yes (e) Yes

EXERCISE 6(a)

1. $\begin{array}{r} -23 \\ - \end{array}$
2. $\begin{array}{r} - \\ - \end{array}$
3. $\begin{array}{r} -9 \\ - \end{array}$
4. $\begin{array}{r} 2 \\ 20 \end{array}$
5. $\begin{array}{r} -9 \\ 19 \end{array}$
6. $\begin{array}{r} 9 \\ 25 \end{array}$
7. $\begin{array}{r} 17 \\ 21 \end{array}$
8. 0
9. $\begin{array}{r} -3 \\ 11 \end{array}$
10. 1
11. $\begin{array}{r} 15 \\ 17 \end{array}$

EXERCISE 6(b)

1. 21 24
2. 57
3. 75 km/hr , 80 km/hr
4. 18 km/hr
5. $\begin{array}{r} 1 \\ 4 \end{array}$

6. Meera = 10 years,
Mother = 40 years

7. 62

8. Length = 14 cm, Breadth = 10 cm

9. Base = 12 cm, Altitude = 20 cm

10. Iron = 50 cu. cm,
Copper = 100 cu. cm

11. 50 hectares, 700 hectares

12. 1411 , 1079

13. Rs 60, Rs 50, Rs 40

14. 40° , 50° , 90°

EXERCISE 7(a)

1. Rs 307.50
2. Rs 1050.00
3. Rs 51.00
4. Rs 249.73
5. Rs 4167.00
6. Rs 202.50
7. Rs 331.00
8. Rs 1261.00
9. Rs 1747.20
10. Rs 6781.25

EXERCISE 7(b)

1. Rs 3307.50
2. Rs 6050
3. Rs 676
4. Rs 2249.73

5. ✓ Rs 12167.00

6. ✓ Rs 4202.50

7. ✓ Rs 1331.00

8. ✓ Rs 9261.00

9. ✓ Rs 4147.20

10. ✓ Rs 22781.25

EXERCISE 7(c)

1. Rs 1020.10

2. Rs 289.00

3. Rs 9261.00

4. Rs 4913.00

5. Rs 231.70

6. Rs 4921.00

7. Rs 8170.00

8. Rs 9826.00

EXERCISE 7(d)

1. Rs 1000.00

2. Rs 4000.00

3. Rs 4096.00

4. 2 years

5. $1\frac{1}{2}$ years

6. 10%

7. 14.4% (approx.)

8. 12.024%

9. Rs 1261.00

10. Rs 1000 ; 10%

EXERCISE 7(e)

1. Rs 72900.00

2. Rs 14440.00

3. 640

4. 132651

5. 2648

6. Rs 13130.00

7. Rs 7938.00

8. 3591

9. 2.853%

10. 80.725%

11. 0.842%

EXERCISE 7(f)

1. Rs 46793.45

2. Rs 2059.20

3. Rs 93586.90

4. 64.345%

EXERCISE 7(g)

1. 2364.4664 lakh tonne

2. 10.308%

3. 31.826%

4. 20.5%

5. 320000 years ; 76000 years

6. 8.673%

7. 8.677476×10^7

ANSWERS

EXERCISE 8(a)

1. Rs 10279.78
2. Rs 8758.12
3. Rs 50.00
4. 108
5. Rs 6575.31
6. Rs 5508.72
7. Rs 10493.60
8. Rs 3190.06

EXERCISE 9(a)

1. Rs 60000.00
2. Re 1 ; Rs 500
3. Rs 960.00
4. Rs 320000, Rs 6000
5. Rs 270000.00
6. 120%

7. 5%
8. 1000
9. Rs 375.00
10. Rs 453995 , 2.75%

EXERCISE 9(b)

1. Rs 18.75
2. Rs 50.00
3. Rs 1150.00
4. 7.14%
5. Second is better
6. Second is better
7. Rs 7717.50-
8. Loss Rs 430.00
9. Rs 27608.00
10. Rs 2000 in shares and Rs 3300 in debentures.

7717

7500.00
217.50

7717.50



Rum aa sika do dawastraga

32412, 2m

num num

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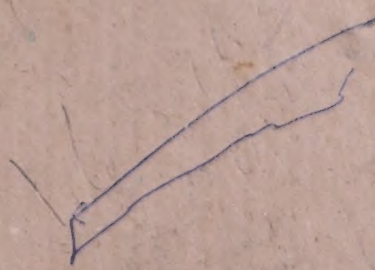
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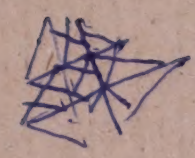
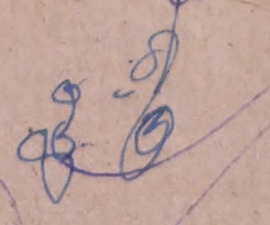
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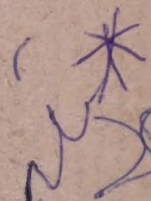
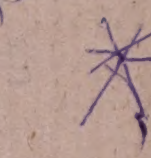


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460



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2 2 5



844
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16 x 84
C 5

